Notes

A2: Solving equations with more than one radical

Solve. Check your solution.

a) $\sqrt{7x+5}=\sqrt{11-3x}$

$(\sqrt{7x+5})^2=(\sqrt{11-3x})^2$

$7x+\cancel{5} = 11-\cancel{3x}$

$+3x\ \cancel{-5} \quad -5\ \cancel{+3x}$

$10x = 6$

$x = \frac{6}{10}$

$\boxed{x = \frac{3}{5}}$ ✓

More than 1 Radical

① Isolate one radical (get it alone on one side)

② Eliminate the radicals

③ Solve for x.

④ Check for extraneous for even roots.

even root

Check:

$\sqrt{\frac{7}{1}(\frac{3}{5})+5} = \sqrt{11-\frac{3}{1}(\frac{3}{5})}$

$\sqrt{\frac{21}{5}+\frac{5}{1}} = \sqrt{\frac{11}{1}-\frac{9}{5}}$

$\sqrt{\frac{21}{5}+\frac{25}{5}} = \sqrt{\frac{55}{5}-\frac{9}{5}}$

$\sqrt{\frac{46}{5}} = \sqrt{\frac{46}{5}}$ ✓

b) $\sqrt[3]{\frac{1}{3}x-6}-\sqrt[3]{2x+14}=0$

• Isolate a radical.

$$\left(\sqrt[3]{\tfrac{1}{3}x-6}\right)^3=\left(\sqrt[3]{2x+14}\right)^3$$

$$\tfrac{1}{3}x-6 = 2x+14$$
$$+6 \qquad\qquad +6$$

$$\cancel{3}\cdot\left(\tfrac{1}{\cancel{3}}x\right)=(2x+20)\cdot 3$$

$$x = \cancel{6x}+60$$
$$-6x \qquad \cancel{-6x}$$

$$-5x = 60$$

$$x=\frac{60}{-5}$$

odd root → no extraneous $\boxed{x=-12}$

Solve. Check your solution.

c) $2\sqrt{x}=\sqrt{x+9}$

$$(2\sqrt{x})^2 = (\sqrt{x+9})^2$$

$$(2)^2\cdot(\sqrt{x})^2 = x+9$$

$$4x = x+9$$

$$-x \quad -x$$

$$3x = 9$$

$$\boxed{x=3}$$

$(2\sqrt{x})^2$ means...

$(2\sqrt{x})\cdot(2\sqrt{x})$

$4\sqrt{x^2}$

$4x$

even root

so Check: $2\sqrt{3} = \sqrt{3+9}$

$$2\sqrt{3} = \sqrt{12}$$

simplify

$2 \lfloor 12$

$2 \lfloor 6$

3

$$2\sqrt{3} = 2\sqrt{3}$$

✓

d) $\sqrt[4]{x-11}=\sqrt[4]{5+2x}$

$$\left(\sqrt[4]{x-11}\right)^4=\left(\sqrt[4]{5+2x}\right)^4$$

$$\cancel{x}-11 = \cancel{5}+2x$$
$$-\cancel{x}\ -5 \quad -\cancel{5} \quad -x$$

$$-16 \cancel{=} x$$

$\boxed{\text{no solution}}$

even root

<u>check</u>: $\sqrt[4]{-16-11}=\sqrt[4]{5+2(-16)}$

even $\sqrt[4]{-27}=\sqrt[4]{-27}$

negative

nonreal

Recall:

a) $x^2 = 4$

even root $\sqrt{x^2} = \sqrt{4}$

$|x| = 2$

$\boxed{x = \pm 2}$ 2 solutions

b) $x^3 = -8$

odd $\sqrt[3]{x^3} = \sqrt[3]{-8}$

$x = -\sqrt[3]{8}$

$\boxed{x = -2}$ 1 solution

Equations with rational exponents

a) Example with one solution

odd (one solution)

$$\frac{3x^{3/2}}{3} = \frac{81}{3}$$

even root

$x^{3/2} \rightarrow (\sqrt{x})^3$

check answers for extraneous

$x^{3/2} = 27$ ← or →

$(x^{3/2})^{2/3} = (27)^{2/3}$

$x = (\sqrt[3]{27})^2$

$x = (3)^2$

$\boxed{x = 9}$

$x^{3/2} = 27$

$(\sqrt{x})^3 = 27$

$(\sqrt{x^3})^2 = (27)^2$

odd root $\sqrt[3]{x^3} = \sqrt[3]{27^2}$

$x = (\sqrt[3]{27})^2$

$x = (3)^2$

$\boxed{x = 9}$

even root so check...

$3(9)^{3/2} = 81$

$3 \cdot (\sqrt{9})^3 = 81$

$3 \cdot (3)^3 =$

$3 \cdot 27 =$

$81 = 81$ ✓

b) Example with two solutions

even (2 solutions, add ±)

$$4x^{2/3} = 64$$
$$\frac{4x^{2/3}}{4} = \frac{64}{4}$$

$x^{2/3} \rightarrow (\sqrt[3]{x})^2$ odd root

no extraneous

$x^{2/3} = 16$ ⟵ or ⟶ $x^{2/3} = 16$

$$(x^{2/3})^{3/2} = \pm(16)^{3/2}$$
$$x = \pm(\sqrt{16})^3$$
$$x = \pm(4)^3$$
$$\boxed{x = \pm 64}$$

$$(\sqrt[3]{x})^2 = 16$$
$$(\sqrt[3]{x^2})^3 = (16)^3$$
$$x^2 = 16^3$$

even root

$$\sqrt{x^2} = \sqrt{16^3}$$
$$|x| = (\sqrt{16})^3$$
$$|x| = (4)^3$$
$$|x| = 64$$
$$\boxed{x = \pm 64}$$

c) $\frac{1}{5}x^{4/3}=125$ even (2 solutions, add $\pm$)

$x^{4/3} \rightarrow \left(\sqrt[3]{x}\right)^4$ odd root

no extraneous

$\cancel{5}\cdot\frac{1}{\cancel{5}}x^{4/3} = 125\cdot 5$

$x^{4/3} = 625$ ← or → $x^{4/3} = 625$

$\left(x^{4/3}\right)^{3/4} = \pm(625)^{3/4}$	$\left(\sqrt[3]{x}\right)^4 = 625$
$x = \pm\left(\sqrt[4]{625}\right)^3$	$\left(\sqrt[3]{x^4}\right)^3 = (625)^3$
$x = \pm(5)^3$	$x^4 = 625^3$
$\boxed{x = \pm 125}$	even root $\sqrt[4]{x^4} = \sqrt[4]{625^3}$
	$\lvert x\rvert = \left(\sqrt[4]{625}\right)^3$
	$\lvert x\rvert = (5)^3$
	$\lvert x\rvert = 125$
	$\boxed{x = \pm 125}$

Odd (1 solution)

d) $\frac{1}{3}x^{3/4} = 9$

$x^{3/4} \rightarrow \underset{\text{even root}}{(\sqrt[4]{x})^3}$

check answers

$\frac{\cancel{3}}{1} \cdot \frac{1}{\cancel{3}} x^{3/4} = 9 \cdot 3$

$x^{3/4} = 27$

$\left(x^{3/4}\right)^{4/3} = (27)^{4/3}$

$x = \left(\sqrt[3]{27}\right)^4$

$x = (3)^4$

$\boxed{x = 81}$

even root so check...

$\frac{1}{3}(81)^{3/4} = 9$

$\frac{1}{3} \cdot \left(\sqrt[4]{81}\right)^3 = 9$

$\frac{1}{3} \cdot (3)^3 = 9$

$\frac{1}{3} \cdot \frac{27}{1} = 9$

$\frac{27}{3} = 9$ ✓

Odd (one solution)

e) $(x-4)^{3/2}+9=1$

$-9 \quad -9$

$(x-4)^{3/2} \rightarrow \left(\sqrt{x-4}\right)^3$

even root

check answers

$(x-4)^{3/2} = -8$

$\left[(x-4)^{3/2}\right]^{2/3} = (-8)^{2/3}$

$x-4 = \left(\sqrt[3]{-8}\right)^2$

$x-4 = (-2)^2$

$x-4 = 4$

$+4 \quad +4$

$\cancel{x=8}$ extraneous

$\boxed{\text{no solution}}$

even root so check...

$(8-4)^{3/2}+9=1$

$(4)^{3/2}+9=1$

$\left(\sqrt{4}\right)^3+9=1$

$(2)^3+9=1$

$8+9=1$

$17 \neq 1$

even (2 solutions, add $\pm$)

f) $1+(3x-7)^{4/3}=17$

$-1 \qquad\qquad -1$

odd root

$(3x-7)^{4/3} \rightarrow \left(\sqrt[3]{3x-7}\right)^4$

no extraneous

* next page

$(3x-7)^{4/3} = 16$

$\left[(3x-7)^{4/3}\right]^{3/4} = \pm(16)^{3/4}$

$3x-7 = \pm\left(\sqrt[4]{16}\right)^3$

$3x-7 = \pm(2)^3$

$3x-7 = \pm 8$

$3x-7=-8$

$+7 \qquad +7$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{-1}{3}$

$\boxed{x=-\frac{1}{3}}$

$3x-7=+8$

$+7 \qquad +7$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{15}{3}$

$\boxed{x=5}$

or

* $$(3x-7)^{4/3} = 16$$

$$\left(\sqrt[3]{3x-7}\right)^4 = 16$$

$$\left(\sqrt[3]{(3x-7)^4}\right)^3 = (16)^3$$

$$(3x-7)^4 = 16^3$$

even

$$\sqrt[4]{(3x-7)^4} = \sqrt[4]{16^3}$$

$$|3x-7| = \left(\sqrt[4]{16}\right)^3$$

$$|3x-7| = (2)^3$$

$$|3x-7| = 8$$

$$3x-7 = \pm 8$$

$$3x-7=-8 \qquad \Big| \qquad 3x-7=+8$$

etc...

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