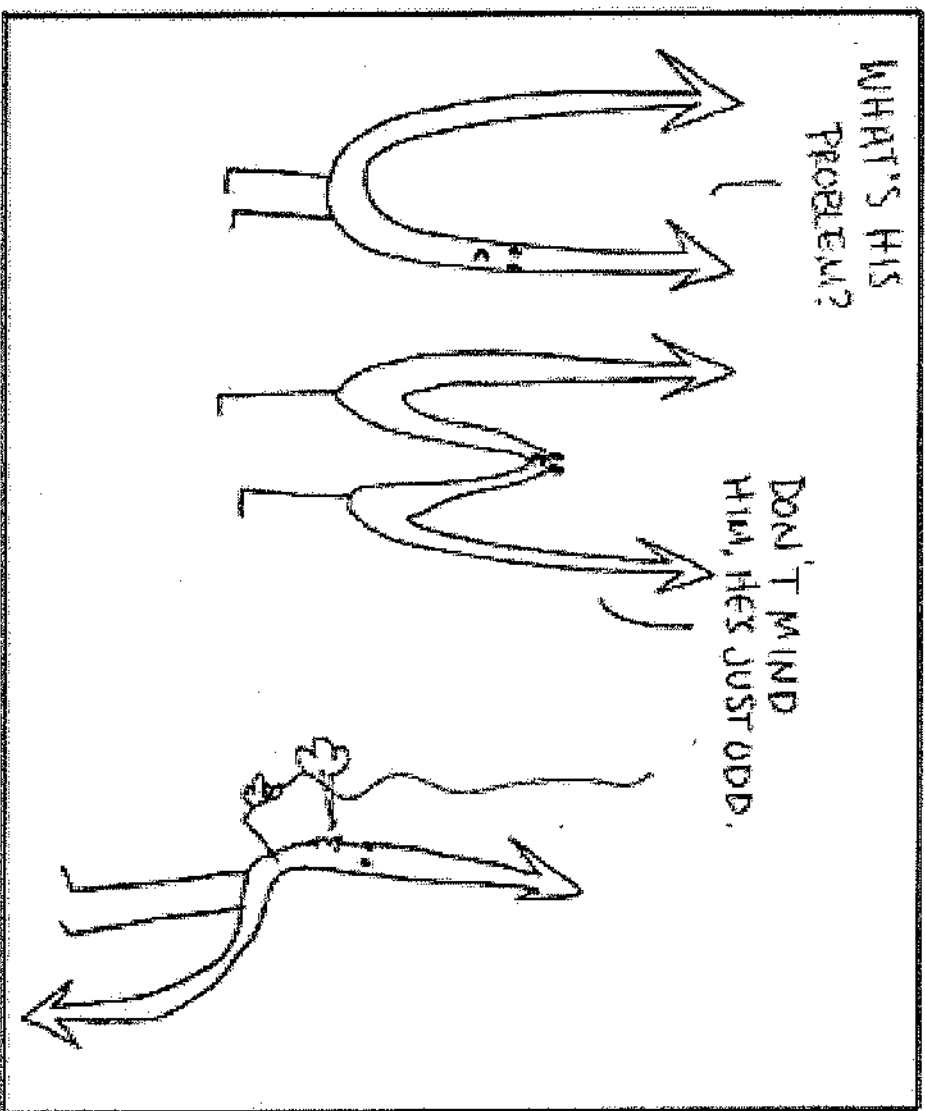


Radical Equations

Day 1



Notes

Square Root Equations - 1 Root

ex: Solve. (REAL SOLUTIONS ONLY)

$$a) \sqrt{x-5} + 2 = 7$$

$$-2 \quad -2$$

$$(\sqrt{x-5})^2 = (5)^2$$

Square both sides

$$x-5 = 25$$

$$\boxed{x = 30}$$

Check in the original question:

$$\begin{aligned} \sqrt{x-5} + 2 &= 7 \\ \sqrt{30-5} + 2 &= 7 \\ \sqrt{25} + 2 &= 7 \\ 5 + 2 &= 7 \quad \checkmark \end{aligned}$$

① Isolate the

root (radical).

② Square both

sides (for square roots)

③ Solve for x .

* ④ Check for

extraneous solutions

(if the root is even)

ex: Solve.

$$b) 3\sqrt{x+2} - 4 = -10$$
$$+4 \quad +4$$

Isolate
radical

$$\frac{\cancel{3}\sqrt{x+2}}{\cancel{3}} = \frac{-6}{3}$$

$$\sqrt{x+2} = -2$$

Square
both
sides

$$(\sqrt{x+2})^2 = (-2)^2$$

$$x+2 = 4$$

$$\cancel{x=2}$$

extraneous

no solution

check:

$$3\sqrt{x+2} - 4 = -10$$

$$3\sqrt{2+2} - 4 = -10$$

$$3\sqrt{4} - 4 = -10$$

$$3(2) - 4 = -10$$

$$6 - 4 = -10$$

$$2 \neq -10$$

ex: solve.

$$c) 1 - 2\sqrt{x^2 - 5x + 15} = -5$$

$$\begin{aligned} \sqrt{x^2 - 5x + 15} &= -6 \\ -\sqrt{x^2 - 5x + 15} &= 6 \end{aligned}$$

$$\sqrt{x^2 - 5x + 15} = 3$$

$$(\sqrt{x^2 - 5x + 15})^2 = (3)^2$$

$$x^2 - 5x + 15 = 9$$

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3) = 0$$

$x=2$	$x=3$
-------	-------

← check each

Check:

$$x=2 \quad 1 - 2\sqrt{(2)^2 - 5(2) + 15} = -5$$

$$1 - 2\sqrt{4 - 10 + 15} = -5$$

$$1 - 2\sqrt{9} = -5$$

$$1 - 2(3) = -5$$

$$1 - 6 = -5$$

$$-5 = -5 \checkmark$$

$$x=3 \quad 1 - 2\sqrt{(3)^2 - 5(3) + 15} = -5$$

$$1 - 2\sqrt{9 - 15 + 15} = -5$$

$$1 - 2\sqrt{9} = -5$$

$$1 - 2(3) = -5$$

$$1 - 6 = -5$$

$$-5 = -5 \checkmark$$

ex: Solve.

d) $\sqrt{x+18} = x-2$

$$(\sqrt{x+18})^2 = (x-2)^2$$

$$x+18 = (x-2)(x-2)$$

FOIL

$$x+18 = x^2 - 2x - 2x + 4$$

$$x+18 = x^2 - 4x + 4$$
$$-x - 18 \quad -x \quad -18$$

$$0 = x^2 - 5x - 14$$

$$0 = (x+2)(x-7)$$

~~$x = -2$~~ $x = 7$
Extraneous check each

Check:

$$x = -2$$

$$\sqrt{-2+18} = -2-2$$

$$\sqrt{16} = -4$$

$$4 \neq -4$$

$$x = 7$$

$$\sqrt{7+18} = 7-2$$

$$\sqrt{25} = 5$$

$$5 = 5 \checkmark$$

ex: Solve.

$$e) (4x+5)^{1/2} = x$$

index = 2

$$\sqrt{4x+5} = x$$

$$(\sqrt{4x+5})^2 = (x)^2$$

$$4x+5 = x^2$$

$$0 = x^2 - 4x - 5$$

$$0 = (x+1)(x-5)$$

$$\cancel{x = -1} \quad \boxed{x = 5}$$

extraneous
check each

check:

$$x = -1$$

$$(4(-1)+5)^{1/2} = -1$$

$$(-4+5)^{1/2} = -1$$

$$(1)^{1/2} = -1$$

$$\sqrt{1} = -1$$

$$1 \neq -1$$

$$x = 5$$

$$(4(5)+5)^{1/2} = 5$$

$$(20+5)^{1/2} = 5$$

$$(25)^{1/2} = 5$$

$$\sqrt{25} = 5$$

$$5 = 5 \quad \checkmark$$

ex: solve.

$$D) 2(x-50)^{1/3} = -10$$

*

odd root

$$\frac{2 \cdot \sqrt[3]{x-50}}{2} = \frac{-10}{2}$$

$$\sqrt[3]{x-50} = -5$$

$$(\sqrt[3]{x-50})^3 = (-5)^3$$

$$x-50 = -125$$
$$+50 \quad +50$$

$$\boxed{x = -75}$$

cube
with
both
sides

* check
odd root,
no extraneous
on these.

$$2(-75-50)^{1/3} = -10$$

$$2(-125)^{1/3} =$$

$$2\sqrt[3]{-125} =$$

$$-2\sqrt[3]{125} =$$

$$-2(5) =$$

$$-10 = -10 \checkmark$$

Nth Root Equations

ex: Solve.

9) $\sqrt[3]{2x+1} - 4 = 1$
+4 +4

Odd root
 $\sqrt[3]{2x+1} = 5$

Isolate radical
 $(\sqrt[3]{2x+1})^3 = (5)^3$

Cube both sides
 $2x+1 = 125$
 -1

$2x = 124$
 $\frac{2x}{2} = \frac{124}{2}$

$x = 62$

ex: Solve.

check:

$$h) \sqrt[4]{5x^2 - 4} = x$$

$$x = -1$$

$$\sqrt[4]{5(-1)^2 - 4} = -1$$

$$\sqrt[4]{5(1) - 4} =$$

$$\sqrt[4]{5-4} =$$

$$\sqrt[4]{1} =$$

$$1 \neq -1$$

even root

$$(\sqrt[4]{5x^2 - 4})^4 = (x)^4$$

$$5x^2 - 4 = x^4$$

$$0 = x^4 - 5x^2 + 4$$

$$0 = (x^2 - 1)(x^2 - 4)$$

$$0 = (x+1)(x-1)(x+2)(x-2)$$

$$x = 1$$

$$\sqrt[4]{5(1)^2 - 4} = 1$$

$$\sqrt[4]{5(1) - 4} =$$

$$\sqrt[4]{5-4} =$$

$$\sqrt[4]{1} =$$

$$1 = 1 \checkmark$$

~~$x = -1$~~ $x = 1$

extraneous

~~$x = 2$~~ $x = 2$

extraneous

• Check each

next page →

check

$$x = -2$$

$$\sqrt[4]{5(-2)^2 - 4} = -2$$

$$\sqrt[4]{5(4) - 4} =$$

$$\sqrt[4]{20 - 4} =$$

$$\sqrt[4]{16} =$$

$$2 \neq -2$$

$$x = 2$$

$$\sqrt[4]{5(2)^2 - 4} = 2$$

$$\sqrt[4]{5(4) - 4} =$$

$$\sqrt[4]{20 - 4} =$$

$$\sqrt[4]{16} =$$

$$2 = 2 \checkmark$$