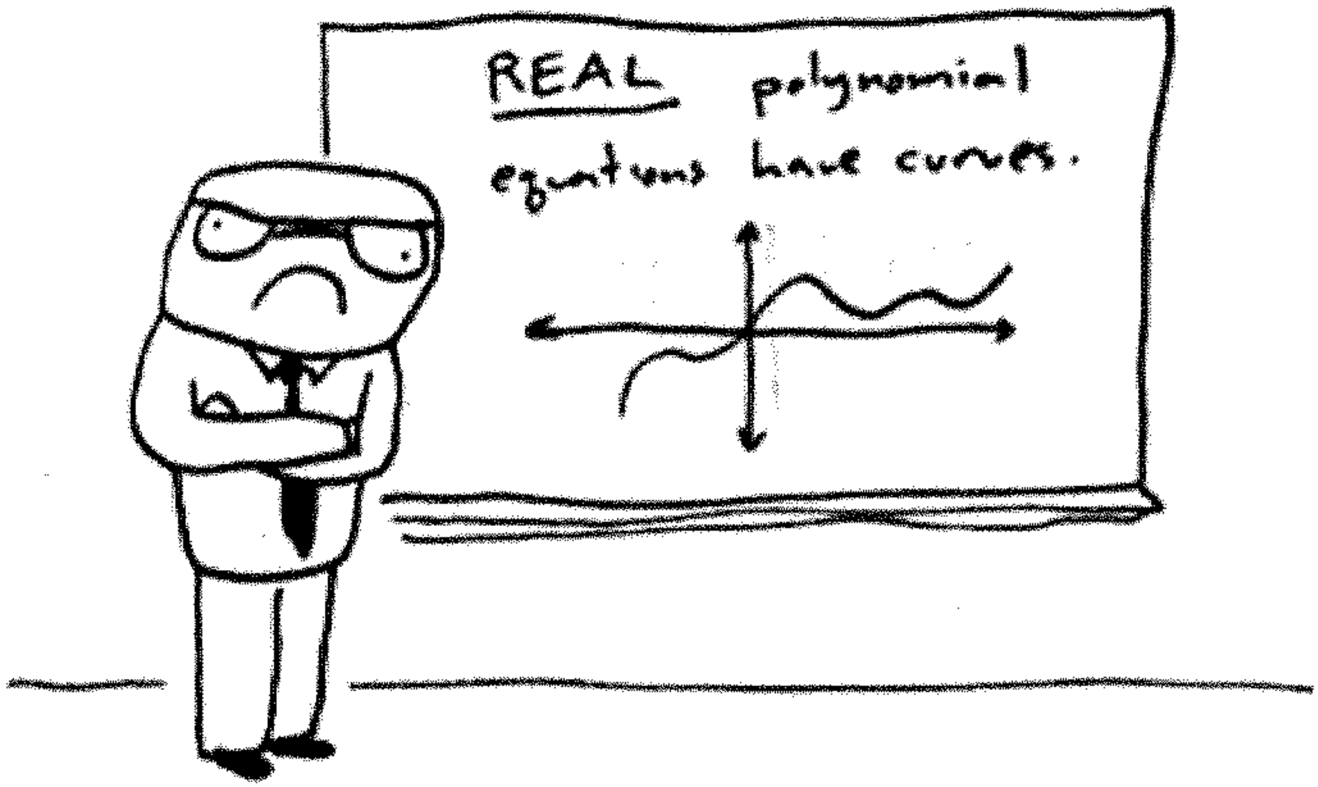
Notes

Solving Polynomial Equations With The RZT



REVIEW

ex: List the possible rational zeros.

$$f(x) = 2x^3 - 7x^2 + 8$$

q p

p		q	
8		2	
1	8	1	2
2	4		

p: $\pm 1, \pm 2, \pm 4, \pm 8$

q: $\pm 1, \pm 2$

$$\boxed{\frac{p}{q} : \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}}$$

(RZT)

ex: Find all zeros using the rational zero theorem.

D3 = 3 answers

a) $f(x) = \overset{q}{x^3} + 7x^2 + 15x + \overset{p}{9}$

$x^2(x+7) + 3(5x+3)$

Does not factor → use RZT

p	q
9	1
1 9 / 3 3	

First:
List Possible Rational zeros

p: ±1, ±3, ±9

q: ±1

$\boxed{\frac{p}{q}: \pm 1, \pm 3, \pm 9}$

Try each until you find one where the R = 0

no MT's

$$\begin{array}{r|rrrr}
-1 & 1 & +7 & +15 & +9 \\
 & \downarrow & -1 & -6 & -9 \\
\hline
D2 \rightarrow & 1 & +6 & +9 & 0 \leftarrow R
\end{array}$$

$\boxed{x = -1}$ $(x^2 + 6x + 9) = 0$

$(x+3)(x+3) = 0$

$(x+3)^2 = 0$

$x + 3 = 0$

$\boxed{x = -1}$ $\boxed{x = -3 \text{ mult}: 2}$

ex: Find all zeros using the rational zero theorem.

b) $g(x) = \overset{q}{2}x^3 - x^2 - 16x + \overset{p}{15}$ ← D3 = 3 answers

no MT's

p: 15 → 1, 15; 3, 5

q: 2 → 1, 2

p: ±1, ±3, ±5, ±15

q: ±1, ±2

$$\boxed{\frac{p}{q}: \pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}}$$

Try each in the box until you get a R=0

$$\begin{array}{r|rrrr} 1 & 2 & -1 & -16 & +15 \\ & \downarrow & +2 & +1 & -15 \\ \hline \text{D2} \rightarrow & 2 & +1 & -15 & 0 \leftarrow R \end{array}$$

$\boxed{X=1}$

$(2x^2 + x - 15) = 0$

$(2x-5)(x+3) = 0$

-30: 1, 30; 2, 15

$\frac{2x}{2}$, $\frac{2x}{-5}$, $+6$, $\div 2$, $\frac{x}{+3}$

$2x - 5 = 0$

$2x = 5$

$\boxed{x = \frac{5}{2}}$

$x + 3 = 0$

$\boxed{x = -3}$

$\boxed{x = 1}$

ex: Find all zeros using the rational zero theorem.

c) $y = 2x^3 + 9x^2 - 33x + 14$

p: $\pm 1, \pm 2, \pm 7, \pm 14$

q: $\pm 1, \pm 2$

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 7, \pm 14, \pm \frac{1}{2}, \pm \frac{7}{2}$

$$\begin{array}{r|rrrr} 2 & 2 & 9 & -33 & 14 \\ & \downarrow & 4 & 26 & -14 \\ \hline & 2 & 13 & -7 & ☺ \end{array}$$

$2x^2 + 13x - 7 = 0 \rightarrow (2x-1)(x+7) = 0$

$\boxed{x = \frac{1}{2}, -7, 2}$

ex: Find all zeros using the rational zero theorem.

D3 = 3 answers

d) $f(x) = \overset{q}{x^3} - 9x^2 + 21x - \overset{p}{4}$

no mt's

p	q
4	1
1 4	
2 2	

p: $\pm 1, \pm 2, \pm 4$

q: ± 1

$\boxed{\frac{p}{q} : \pm 1, \pm 2, \pm 4}$

Try each until you find one where the R = 0

$$\begin{array}{r|rrrr} 4 & 1 & -9 & +21 & -4 \\ & \downarrow & +4 & -20 & +4 \\ \hline D2 \rightarrow & 1 & -5 & +1 & 0 \leftarrow R \end{array}$$

$\boxed{X=4}$

$(X^2 - 5x + 1) = 0$

Does not factor

QF

$a=1$
$b=-5$
$c=1$

$X = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(1)}}{2(1)}$

$X = \dfrac{5 \pm \sqrt{25-4}}{2}$

$\sqrt{21}$
3 7

$\boxed{X=4}$ $\boxed{X = \dfrac{5 \pm \sqrt{21}}{2}}$

ex: Determine the best method for finding the zeros of the given polynomial. DO NOT SOLVE.

a) $f(x) = 10x^3 - 17x^2 - 7x + 2$

RZT

b) $f(x) = 16x^4 - 54x$

factor

Factoring or RZT?

Before finding all zeros consider whether the polynomial factors

- if YES, then solve by factoring

- if NO, then solve by RZT.

*If the polynomial is QUADRATIC,
then factor, square roots or quad formula.

Complex Conjugate & Irrational Conjugate Theorem

Imaginary and irrational roots always come in conjugate pairs.

Complex Conjugate and Irrational Conjugate Theorem

Imaginary and irrational roots always come in conjugate pairs

$2i$ & $-2i$ $\quad$ $\sqrt{7}$ & $-\sqrt{7}$

$-6i$ & $6i$ $\quad$ $-\sqrt{3}$ & $\sqrt{3}$

Write a polynomial function, f(x), in standard form with integral coefficients and the given roots.

1) Given: $-2, \sqrt{6}, 0$

you would need to add its conjugate to the list

add to list

Put this one in front

$x = 0 \qquad x = -2 \qquad x = \sqrt{6} \qquad x = -\sqrt{6}$

$(x+2) = 0 \qquad (x-\sqrt{6}) = 0 \qquad (x+\sqrt{6}) = 0$

$$f(x) = x\,(x+2)\Big[\underbrace{(x-\sqrt{6})(x+\sqrt{6})}_{\text{FOIL}}\Big]$$

$$(x^2 + \cancel{x\sqrt{6}} - \cancel{x\sqrt{6}} - \sqrt{36})$$

$$f(x) = x\Big[\underbrace{(x+2)(x^2-6)}_{\text{FOIL}}\Big]$$

$$f(x) = x\,(x^3 - 6x + 2x^2 - 12)$$

$$f(x) = x^4 - 6x^2 + 2x^3 - 12x$$

reorder

$$\boxed{f(x) = x^4 + 2x^3 - 6x^2 - 12x}$$

Write a polynomial function, f(x), in standard form with integral coefficients and the given roots.

2) Given: $4, 5i$

you would need to add its conjugate to the list

add to list

$x = 4$ $\quad x = 5i$ $\quad x = -5i$

$(x-4)=0$ $\quad (x-5i)=0$ $\quad (x+5i)=0$

$$f(x) = (x-4)\left[\underbrace{(x-5i)(x+5i)}_{\text{FOIL}}\right]$$

$$(x^2 + \cancel{x5i} - \cancel{x5i} - 25(i^2))$$

$$f(x) = \underbrace{(x-4)(x^2+25)}_{\text{FOIL}}$$

$$f(x) = x^3 + 25x - 4x^2 - 100$$

reorder

$$\boxed{f(x) = x^3 - 4x^2 + 25x - 100}$$

ex: Write a polynomial function in standard form with integral coefficients and the given roots.

a) $-\frac{2}{5}$, $3i$, $\overset{\text{add}}{-3i}$

* Are any zeros irrational or imaginary?

$$f(x) = (5x+2)\underbrace{(x-3i)(x+3i)}_{\text{foil}}$$

$$f(x) = (5x+2)(x^2 + \cancel{3ix} - \cancel{3ix} - 9i^2)$$

$$f(x) = \underbrace{(5x+2)(x^2+9)}_{\text{foil}}$$

$$\boxed{f(x) = 5x^3 + 2x^2 + 45x + 18}$$

ex: Write a polynomial function in standard form with integral coefficients and the given roots.

b) o mult 5, $-\sqrt{2}$, $\sqrt{5}$, $\overset{\text{add}}{\sqrt{2}}$, $\overset{\text{add}}{-\sqrt{5}}$

$$f(x) = x^5\left[\underbrace{(x+\sqrt{2})(x-\sqrt{2})}_{\text{foil}}\right]\left[\underbrace{(x-\sqrt{5})(x+\sqrt{5})}_{\text{foil}}\right]$$

$$f(x) = x^5\left[\underbrace{(x^2-2)(x^2-5)}_{\text{foil}}\right]$$

$$f(x) = x^5(x^4 - 7x^2 + 10)$$

$$\boxed{f(x) = x^9 - 7x^7 + 10x^5}$$

REVIEW

ex: Find the error.

add MT

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -5 & 3 \\ & & & 2 & -6 \\ \hline & 1 & & -3 & -3 \end{array}$$

$$\frac{x^3 - 5x + 3}{x - 2} = x^2 - 3x - \frac{3}{x - 2}$$