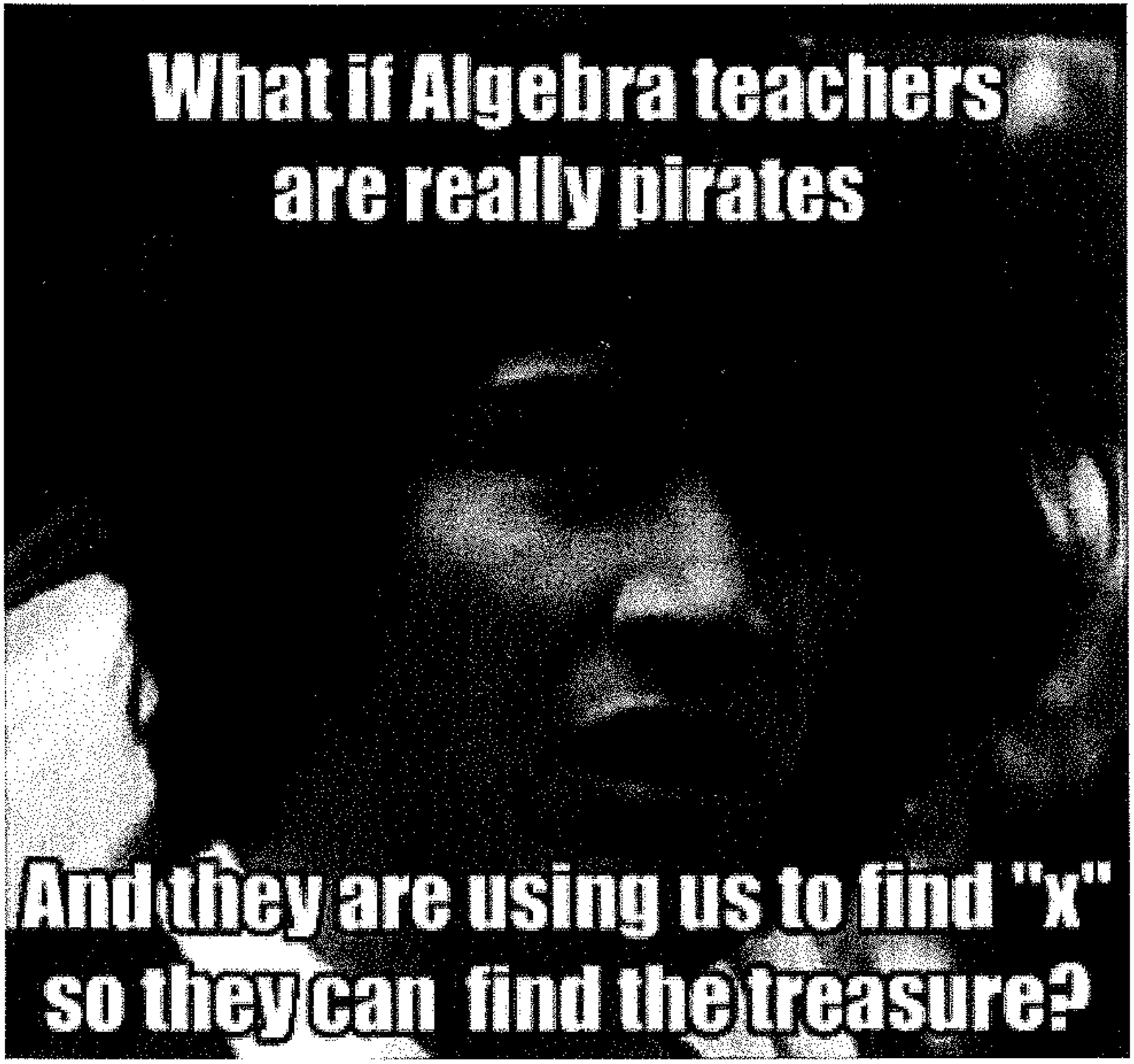
Notes

# A2 - Solving Polynomial Equations by Factoring



Theorem:

largest power on the variable

A polynomial equation with degree n has _n__ solutions.

Vocabulary:

solutions/roots - answers to an equation

zeros - quantities that make a function equal to zero

Find the solutions.

ex: Solve by factoring.

degree 2

a) $x^2 - 8x + 15 = 0$

$+15$

$\frac{x}{-3} \quad \frac{x}{-5}$

$(x-3)(x-5) = 0$

$x - 3 = 0 \quad | \quad x - 5 = 0$

$\boxed{x = 3} \quad | \quad \boxed{x = 5}$

ex: Solve by factoring.

no gcf

degree 4

b) $2x^{4} + 7x^{2} - 15 = 0$

$-30$

$\frac{2x^2}{-3}$ $\frac{2x^2}{+10}$

$\div 2$

$\frac{1x^2}{+5}$

$(2x^2 - 3)(x^2 + 5) = 0$

SR

$2x^2 - 3 = 0$

$2x^2 = 3$

$\sqrt{x^2} = \sqrt{\frac{3}{2}}$

$|x| = \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$|x| = \frac{\sqrt{6}}{2}$

$\boxed{x = \pm \frac{\sqrt{6}}{2}}$

SR

$x^2 + 5 = 0$

$\sqrt{x^2} = \sqrt{-5}$

$|x| = i\sqrt{5}$

$\boxed{x = \pm i\sqrt{5}}$

4 solutions (degree 4)

ex: Solve by factoring.

degree 4

c) $\dfrac{24x^{4}}{3x} + \dfrac{3x}{3x} = 0$

gcf

$3x(8x^3+1)=0$

cubes

cube:
$a = 2x$
$b = 1$

$a^2 = 2x \cdot 2x = 4x^2$
$ab = 2x \cdot 1 = 2x$
$b^2 = 1$

$4 \lfloor 12$
$3$

S O AP

$3x\,(2x+1)(4x^2-2x+1)=0$

$+4$
$1 \quad 4$
$\cancel{2} \quad \cancel{2}$

$\sqrt{-12}$
$i\sqrt{12}$
$i\sqrt{4\cdot 3}$
$2i\sqrt{3}$

$\dfrac{\cancel{3}x}{\cancel{3}} = \dfrac{0}{3}$

$\boxed{x=0}$

$2x+1=0$
$2x=-1$
$\boxed{x=-\frac{1}{2}}$

$4x^2-2x+1=0$
need QF

$a=4$
$b=-2$
$c=1$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2-4(4)(1)}}{2(4)}$

$x = \dfrac{2 \pm \sqrt{4-16}}{8} = \dfrac{2 \pm \sqrt{-12}}{8}$

$x = \dfrac{2 \pm 2i\sqrt{3}}{8}$ simplify...

$\boxed{x = \dfrac{1 \pm i\sqrt{3}}{4}}$

ex: Solve by factoring.

d) $\underbrace{\frac{x^{3}}{x^2} - \frac{5x^2}{x^2}} \underbrace{- \frac{9x}{-9} + \frac{45}{-9}} = 0$ ← degree 3

$x^2(x-5) - 9(x-5) = 0$

$(x-5)(x^2-9) = 0$
DOS

$(x-5)(x+3)(x-3) = 0$

| | | |
|---|---|---|
| $x-5=0$ | $x+3=0$ | $x-3=0$ |
| $\boxed{x=5}$ | $\boxed{x=-3}$ | $\boxed{x=3}$ |

ex: Solve by factoring.

e) $x^4 + 2x^2 + 1 = 0$ degree

$$+1 \qquad \frac{x^2}{+1} \quad \frac{x^2}{+1}$$

$(x^2+1)(x^2+1) = 0$

$(x^2+1)^2 = 0$

$x^2+1 = 0$

SR

$\sqrt{x^2} = \sqrt{-1}$

$|x| = i$

$\boxed{x = \pm i \quad \underline{\text{mult}}: 2}$

$x = \pm i$

ex: Solve by factoring.

f) $\underbrace{x^{7}}_{x^5} - \underbrace{64x^5}_{x^5} = 0$ ← degree 7

gcf

$x^5(x^2 - 64) = 0$

DOS

$x^5(x+8)(x-8) = 0$

$x^5 = 0$ | $x+8=0$ | $x-8=0$

$\boxed{x=0 \text{ mult: } 5}$ | $\boxed{x=-8}$ | $\boxed{x=8}$

$x^5 = 0$

$x \cdot x \cdot x \cdot x \cdot x = 0$

ex: Solve by factoring.

g) $-2x^7 \cdot (x^2-2)^2 \cdot (3x+4) = 0$

$$-2x^7 = 0 \qquad \frac{-2x^7}{-2} = \frac{0}{-2} \qquad x^7 = 0 \qquad \boxed{\begin{array}{c} x = 0 \\ \text{mult: } 7 \end{array}}$$

$$(x^2-2)^2 = 0 \qquad x^2 - 2 = 0$$

SR

$$\sqrt{x^2} = \sqrt{2} \qquad |x| = \sqrt{2} \qquad \boxed{\begin{array}{c} x = \pm\sqrt{2} \\ \text{mult: } 2 \end{array}}$$

$$3x+4 = 0 \qquad 3x = -4 \qquad \boxed{x = \frac{-4}{3}}$$

Degree:

$$x^7 (x^2)^2 \cdot x$$

$$x^7 \cdot x^4 \cdot x^1$$

$$x^{12} \leftarrow \text{degree } 12$$

NO

$$(x^2-2)^2 \neq x^4 - 4$$

$$\neq x^4 + 4$$

Solve.

degree:

$x^1 \cdot x^2 \cdot x^3$

$x^6$

a) $x(x+4)^2(2x-5)^3 = 0$

$\boxed{x=0}$

$(x+4)^2 = 0$

$x+4=0$

$\boxed{x=-4 \quad \text{mult}: 2}$

$(2x-5)^3 = 0$

$2x-5=0$

$2x=5$

$\boxed{x=\frac{5}{2} \quad \text{mult}: 3}$

b) $9x^4 + 2x^2 + 4 = \cancel{3} - \cancel{4x^2}$

$+4x^2 - 3 \quad \cancel{-3} \; \cancel{+4x^2}$

degree 4

$9x^4 + 6x^2 + 1 = 0$

$+9$

$\frac{9x^2}{+3} \quad \frac{9x^2}{+3}$

$\div 3 \quad \div 3$

$\frac{3x^2}{+1} \quad \frac{3x^2}{+1}$

$(3x^2+1)(3x^2+1) = 0$

$(3x^2+1)^2 = 0$

$3x^2+1=0$

SR

$3x^2 = -1$

$\sqrt{x^2} = \sqrt{\frac{-1}{3}}$

$|x| = \frac{\sqrt{-1}}{\sqrt{3}}$

$|x| = \frac{i}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$|x| = \frac{i\sqrt{3}}{3}$

$\boxed{x = \pm \frac{i\sqrt{3}}{3} \quad \text{mult}: 2}$