Notes

Solving polynomial equations (continued)

Write a polynomial equation in standard form with integral coefficients given the roots.

a) $0, -3, 5$ degree 3

$x=0 \quad x=-3 \quad x=5$

$(x+3)=0 \quad (x-5)=0$

$f(x) = x[(x+3)(x-5)]$

Foil

$f(x) = x(x^2-2x-15)$

$\boxed{f(x) = x^3-2x^2-15x}$

b) $\frac{2}{3}, \sqrt{5}, -\sqrt{5}$ degree 3

$x=\frac{2}{3} \quad x=\sqrt{5} \quad x=-\sqrt{5}$

$3 \cdot x = \frac{2}{3} \cdot 3$ $\quad (x-\sqrt{5})=0 \quad (x+\sqrt{5})=0$

$3x=2$

$(3x-2)=0$

multiply the conjugates first

$f(x) = (3x-2)[(x-\sqrt{5})(x+\sqrt{5})]$

Foil

$f(x) = (3x-2)(x^2+x\sqrt{5}-x\sqrt{5}-\sqrt{25})$

$f(x) = (3x-2)(x^2-5)$

Foil

$f(x) = 3x^3-15x-2x^2+10$

reorder

$\boxed{f(x) = 3x^3-2x^2-15x+10}$

Write a polynomial equation in standard form with integral coefficients given the roots.

c) $1, 2i, -2i$ degree 3

$x=1$ $x=2i$ $x=-2i$

$(x-1)=0$ $(x-2i)=0$ $(x+2i)=0$

multiply conjugates first

$f(x)=(x-1)[(x-2i)(x+2i)]$

Foil

$f(x)=(x-1)(x^2+2ix-2ix-4i^2)$

$i^2=-1$

$f(x)=(x-1)(x^2+4)$

Foil

$f(x)=x^3+4x-x^2-4$

reorder

$f(x)=x^3-x^2+4x-4$

d) 3, -4 (mult of 2)

3, -4, -4 degree 3

$x=3$ $x=-4$ $x=-4$

$(x-3)=0$ $(x+4)=0$ $(x+4)=0$

$f(x)=(x-3)[(x+4)(x+4)]$

FOIL

$f(x)=(x-3)(x^2+8x+16)$

$= x^3+8x^2+16x$

$+ \quad -3x^2-24x-48$

$f(x)=x^3+5x^2-8x-48$

Solve.

already factored here!

a) $x(x+4)^2(2x-5)^3=0$

and =0

$X^1 \cdot (X^1)^2 \cdot (X^1)^3$

$X^1 \cdot X^2 \cdot X^3$

$X^{(6)}$ degree 6 → 6 solutions

$X=0$

$(X+4)^{(2)}=0$

$X+4=0$

$X=-4$
mult: 2

$(2X-5)^{(3)}=0$

$2X-5=0$

$2X=5$

$X=\frac{5}{2}$
mult: 3

Solve. SET = 0

b) $9x^4+2x^2+4=3-4x^2$

$+4x^2 \quad -3 \quad -3 \quad +4x^2$

PST $9x^4+6x^2+1=0$ degree 4 $\Rightarrow$ 4 solutions

$(3x^2+1)(3x^2+1)=0$

$(3x^2+1)^{(2)}=0$ ← mult: 2

$3x^2+1=0$

$3x^2=-1$

$\sqrt{x^2}=\sqrt{\frac{-1}{3}}$

$|x|=\frac{\sqrt{-1}}{\sqrt{3}}$

$|x|=\frac{i}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$|x|=\frac{i\sqrt{3}}{\sqrt{9}}$

$|x|=\frac{i\sqrt{3}}{3}$

$\boxed{x=\pm\frac{i\sqrt{3}}{3} \quad \text{mult: } 2}$

Solve.

degree 5

c) $7x^5 - 30x^3 + 8x = 0$

GCF: x x x

$x(7x^4 - 30x^2 + 8) = 0$ — factor quadratic in form

$+56$ $\quad 7x^2$

$\frac{7x^2}{-2}$ $\quad -28$

$\div 7$

$\frac{x^2}{-4}$

$x(7x^2 - 2)(x^2 - 4) = 0$

DOS

$x(7x^2 - 2)(x+2)(x-2) = 0$

$x = 0$

$7x^2 - 2 = 0$

SR

$x = \pm 2$

$7x^2 = 2$

$\sqrt{x^2} = \sqrt{\frac{2}{7}}$

$|x| = \frac{\sqrt{2}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$

$|x| = \frac{\sqrt{14}}{\sqrt{49}}$

$|x| = \frac{\sqrt{14}}{7}$

$x = \pm \frac{\sqrt{14}}{7}$

Solve. set = 0

d) $100x^4 = 64x^2$

$100x^4 - 64x^2 = 0$

GCF: $4x^2$

degree 4 → 4 solutions

$4x^2(25x^2 - 16) = 0$

DOS

$4x^2(5x+4)(5x-4) = 0$

↓

$\frac{4x^2}{4} = \frac{0}{4}$

$x^2 = 0$

$x = 0$
mult: 2

$5x+4=0$

$5x = -4$

$x = -\frac{4}{5}$

$5x-4=0$

$5x = 4$

$x = \frac{4}{5}$

Solve. Already factored!

e) $(5x+4)(2x^2+x-5)=0$

$x^1 \cdot x^2 = x^{(3)}$ degree 3

$5x+4=0 \quad 2x^2+x-5=0$

$5x=-4$ QF

$\boxed{x=-\frac{4}{5}}$ $a=2$, $b=1$, $c=-5$

-10, 1, 10, 2, 5

$$x=\frac{-(1)\pm\sqrt{(1)^2-4(2)(-5)}}{2(2)}$$

$$x=\frac{-1\pm\sqrt{1+40}}{4}$$

$$\boxed{x=\frac{-1\pm\sqrt{41}}{4}}$$

set = 0

f) $2x^3-11=x^3+16$

$-x^3-16 \quad -x^3-16$

degree 3 ⇒ 3 solutions

$x^3-27=0$

cube

$(x-3)(x^2+3x+9)=0$ (S, AP)

QF

$x-3=0$

$\boxed{x=3}$ $a=1$, $b=3$, $c=9$

$$x=\frac{-(3)\pm\sqrt{(3)^2-4(1)(9)}}{2(1)}$$

$$x=\frac{-3\pm\sqrt{9-36}}{2}$$

$\sqrt{-27}$

$i\sqrt{27}$

$$x=\frac{-3\pm\sqrt{-27}}{2}$$

3|27, 3|9, 3

$3i\sqrt{3}$

$$\boxed{x=\frac{-3\pm 3i\sqrt{3}}{2}}$$

or

$$\boxed{x=-\frac{3}{2}\pm\frac{3i\sqrt{3}}{2}}$$