A2 Solving Exponential & Logarithmic Equations Day 2

Notes

Logic: another thing that penguins aren't very good at.

2 Types of Exponential Equations:

1. $a^x = b$, where a and b are integral powers of the same number

$$\text{ex: } 27^x = 9$$

2. $a^x = b$, where a and b are NOT integral powers of the same number

$$\text{ex: } 3^x = 5$$

REVIEW:

ex: Solve.

$$27^{3x-7} = 81^{12-3x}$$

$$(\cancel{3}^3)^{3x-7} = (\cancel{3}^4)^{12-3x}$$

$$3(3x-7) = 4(12-3x)$$

$$9x - \cancel{21} = 48 - \cancel{12x}$$

$$+12x \ \cancel{+21} \qquad +21 \ \cancel{+12x}$$

$$\frac{\cancel{21}x}{\cancel{21}} = \frac{69}{21}$$

Reduces (÷ 3)

$$\boxed{x = \frac{23}{7}}$$

Type 2

ex: Solve. Round to 3 decimal places. where needed.

a) $3^x = 5$ Can't get the same base ... So need to use logs.

$\log 3^x = \log 5$ Take log of each side.

$x \log 3 = \log 5$

$\boxed{x = \frac{\log 5}{\log 3}}$ ← Show this as exact.

or $\boxed{x = \log_3 5}$

$\boxed{x \approx 1.465}$ ← Rounded answer.

approximately

ex: Solve.

b) $e^{x+1} = 10$ When you have a base of $\underline{\underline{e}}$, use "ln".

$\ln e^{(x+1)} = \ln 10$ add Take the natural log of each side.

$(x+1)\underbrace{\ln e}_{1} = \ln 10$

Recall:
$* \ln e = 1$

$x + 1 = \ln 10$
$\quad -1 \quad -1$

$\boxed{x = -1 + \ln 10}$ or $x = \ln \underbrace{10 - 1}$ could be confusing here; do not subtract these.

$\boxed{x \approx 1.303}$

ex: Solve.

c) $3 \cdot 4^{x-7} + \cancel{6} = 54$

$$\quad\;\; -\cancel{6} \quad -6$$

$$\frac{\cancel{3} \cdot 4^{x-7}}{\cancel{3}} = \frac{48}{3}$$

$$4^{x-7} = 16 \quad \leftarrow \text{non calc... no logs needed.}$$

$$\cancel{4}^{(x-7)} = (\cancel{4}^{2})$$

$$x - 7 = 2$$

$$\;\; +7 \quad +7$$

$$\boxed{x = 9}$$

ex: Solve.

d) $2 \cdot 10^{x-3} - 3 = 37$

$\quad\quad\quad\quad +3 \quad +3$

$\dfrac{\cancel{2} \cdot 10^{x-3}}{\cancel{2}} = \dfrac{40}{2}$

$10^{x-3} = 20$

Can't get same base, need logs..

$\log 10^{(x-3)} = \log 20$

$(x-3)\underbrace{\log_{10} 10}_{1} = \log 20$

$x - 3 = \log 20$ or $x = \log(20) + 3$

$\quad +3 \quad\; +3$

$\boxed{x = 3 + \log 20}$

$\boxed{x \approx 4.301}$

ex: Solve.

e) $\cancel{2} - 5^{x-2} = 3$

$\quad -\cancel{2} \qquad\qquad -2$

$$-5^{x-2} = 1$$

$$\frac{\cancel{-1} \cdot 5^{x-2}}{\cancel{-1}} = \frac{1}{-1}$$

$$5^{x-2} = -1$$ ← can't get the same base

$$\log 5^{x-2} = \log(-1)$$

not possible

$\boxed{\text{no solution}}$

REVIEW: Evaluate. NON-CALC

a) $\log_3\left(\frac{1}{9}\right) = \log_3(3^{-2}) = \boxed{-2}$

b) $\frac{\log 36}{\log 6}$ use change of base rule $= \log_6 36 = \log_6(6^2) = \boxed{2}$

c) $\log_8(-8) = \boxed{\text{undefined}}$

not possible

d) $\log 0 = \boxed{\text{undefined}}$

not possible

Domain of Logarithmic Functions

$$y = \log_b\left(f(x)\right)$$

Domain: $f(x) > 0$

ex: State the domain in set notation.

a) $y = \log_2(x-1)$

$$x - 1 > 0$$

$$x > 1$$

$$\boxed{\{x \mid x > 1\}}$$

ex: State the domain in set notation.

b) $y = 3 - \ln(-x)$

$$\frac{-x}{-1} > \frac{0}{-1}$$

flip

$$x < 0$$

$$\boxed{\{x \mid x < 0\}}$$

c) $y = \log_9 (x^2 + 9)$

greater than 1, make an equation and number line to solve.

$$x^2 + 9 > 0$$

$$x^2 + 9 = 0$$

$$\sqrt{x^2} = \sqrt{-9}$$

$$|x| = 3i$$

$$x = \pm 3i$$

imaginary ... no restriction on Reals.

not sure, try plugging in 0, positive #'s, and negative #'s

so... All real #'s

$$\boxed{\{x \mid x \in R\}}$$

Solving Logarithmic Equations

Two Types:

1. 1 Logarithm

   ex: $2 - \log_2(x+1) = 4$

2. More than 1 Logarithm

   ex: $\log_3(x^2 - 3) = \log_3 2 + \log_3 x$

When solving type 1 or 2, rewrite the equation with ONE TERM on each side of the equation.

ex: Solve.

a) $\log_{36} x + \frac{1}{2} = 2$

$\quad\quad\quad\quad -\frac{1}{2} \quad -\frac{1}{2}$

$\log_{36} x = 1\frac{1}{2}$ ← Rewrite as improper fraction.

$\log_{36} x = \frac{3}{2}$ • change to exponential form

$(36)^{3/2} = x$

$x = (\sqrt{36})^3$

$x = (6)^3$

$\boxed{x = 216}$ ✓ • check to make sure the argument remains positive.

ex: Solve.

b) $\log_{12}\left(x^2-4\right)=\log_{12}\left(3x\right)$

- already has one term on each side.

$$\cancel{\log_{12}}(x^2-4) = \cancel{\log_{12}}(3x)$$

Quadratic $\rightarrow$

$$x^2-4 = 3\cancel{x}$$
$$-3x \qquad -\cancel{3x}$$

set $=0$ $\rightarrow$

$$x^2-3x-4 = 0$$
$$(x-4)(x+1) = 0$$

$x-4=0$ | $x+1=0$

$\boxed{x=4}$ ✓ | $\cancel{x=-1}$ extraneous

check each in the original logs' arguments, make sure each remains positive.

ex: Solve.

c) $\log_{31}(4x-5)-\log_{31}(2x-1)=0$

$$\cancel{\log_{31}}(4x-5) = \cancel{\log_{31}}(2x-1)$$

$$\begin{aligned} 4x-\cancel{5} &= 2\cancel{x}-1 \\ -2x+\cancel{5} &\quad -\cancel{2}x+5 \end{aligned}$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{4}{2}$$

$$\boxed{x=2}$$

check in the argument

ex: Solve.

d) $\log_2(4x-5)-\log_2(2x-1)=3$

• condense

$$\log_2\left[\frac{4x-5}{2x-1}\right] = 3$$

• now change to exponential form

$$2^3 = \frac{4x-5}{2x-1}$$

• cross multiply

$$\frac{8}{1} = \frac{4x-5}{2x-1}$$

$$8(2x-1) = 4x-5$$

$$16x-\cancel{8} = \cancel{4x}-5$$
$$-4x+\cancel{8} \quad -\cancel{4x} \quad +8$$

$$\frac{\cancel{12}x}{\cancel{12}} = \frac{3}{12}$$

$$\cancel{x = \frac{1}{4}}$$

extraneous

check

Check:

$4x-5$

$\cancel{4}\left(\frac{1}{\cancel{4}}\right)-5$

$1-5$

-4 X

$\boxed{\text{no solution}}$

ex: Solve.

e) $\underbrace{\log 18 - \log 3x}$ $= \log 2$

- condense

$$\cancel{\log}\left[\frac{18}{3x}\right] = \cancel{\log}\, 2$$

$$\frac{18}{3x} = \frac{2}{1}$$

- cross multiply

$$\frac{\cancel{6}x}{\cancel{6}} = \frac{18}{6}$$

$$\boxed{x = 3} \checkmark$$

check in the argument.

ex: Solve.

f) $\dfrac{\ln(-x)}{2} + \cancel{4} = 5$

$\quad\quad\quad\;\; \cancel{-4} \quad -4$

$\cancel{2} \cdot \dfrac{\ln(-x)}{\cancel{2}} = 1 \cdot 2$

$\ln(-x) = 2$ • change to exponential form

$\log_e(-x) = 2$

$\dfrac{e^2}{-1} = \dfrac{-x}{-1}$

$\boxed{x = -e^2}$ ✓

w/calc. $\boxed{x \approx -7.389}$

Check: $\ln(-x)$

$\ln(+(+e^2))$

$\ln(e^2)$

✓

ex: Solve.

g) $2\log x = \underbrace{\log 2 + \log 8}_{\text{• condense}}$

$$2\log x = \log(2\cdot 8)$$

$$2\log x = \log 16$$

$$\cancel{\log} x^2 = \cancel{\log} 16$$

$$\sqrt{x^2} = \sqrt{16}$$

$$|x| = 4$$

$$x = \pm 4$$

extraneous

$\cancel{x = -4}$ $\boxed{x = 4}$

check each.

ex: Solve.

h) $\log_5 \sqrt{x-2} = 1$

- change to exponential form.

$$5^1 = \sqrt{x-2}$$

$$(5)^2 = \left(\sqrt{x-2}\right)^2$$

- square both sides.

$$25 = x - \cancel{2}$$

$$+2 \qquad\qquad +\cancel{2}$$

$$\boxed{x = 27}$$ ✓

check it.

MIXED PRACTICE

ex: Solve.

a) $5 \cdot 2^{x} - 3 = 157$

b) $8^{x+1} = 4^{x-3}$

c) $9^{3x^2-6x} = 3^{x+5}$

challenge

* d) $6^{3-6x} = 3^{x+5}$

a) $5 \cdot 2^x - \cancel{3} = 157$

$\cancel{+3} \quad +3$

non-calc.

$$\frac{\cancel{5} \cdot 2^x}{\cancel{5}} = \frac{160}{5}$$

$$2^x = 32$$

$$2^x = 2^5$$

$$\boxed{x = 5}$$

b) $8^{x+1} = 4^{x-3}$

non-calc

$$(\cancel{2}^3)^{(x+1)} = (\cancel{2}^2)^{(x-3)}$$

$$3(x+1) = 2(x-3)$$

$$3x + \cancel{3} = \cancel{2x} - 6$$
$$-2x - \cancel{3} \qquad -\cancel{2x} - 3$$

$$\boxed{x = -9}$$

c) $9^{3x^2-6x} = 3^{x+5}$

$(\cancel{3}^2)^{(3x^2-6x)} = \cancel{3}^{(x+5)}$

$2(3x^2-6x) = x+5$

$6x^2-12x = \cancel{x}+5$

$\quad -x \qquad \cancel{-x}$

$6x^2-13x = \cancel{5}$

$\quad -5 \qquad \cancel{-5}$

no gcf

$6x^2-13x-5=0$

-30

$\cancel{1 \quad 30}$

$\frac{6x}{+2}$ $\frac{6x}{-15}$

$\div 2$ $\div 3$

$\frac{3x}{+1}$ $\frac{2x}{-5}$

$(3x+1)(2x-5)=0$

$3x+1=0$	$2x-5=0$
$3x=-1$	$2x=5$
$\boxed{x=-\frac{1}{3}}$	$\boxed{x=\frac{5}{2}}$

challenge

★ d) $6^{3-6x} = 3^{x+5}$ • can't get the same base, need "logs".

$\log 6^{(3-6x)} = \log 3^{(x+5)}$

$(3-6x)\log 6 = (x+5)\log 3$ • distribute each "log".

$3\log 6 - 6x\log 6 = x\log 3 + 5\log 3$ ← • get all the "logs" with an "x" on one side, and all the "logs" without an "x" on the other side.

$-5\log 3 + 6x\log 6 \qquad +6x\log 6 - 5\log 3$

$3\log 6 - 5\log 3 = x\log 3 + 6x\log 6$

• factor out an "x"

$\frac{3\log 6 - 5\log 3}{\log 3 + 6\log 6} = \frac{x(\cancel{\log 3 + 6\log 6})}{\cancel{\log 3 + 6\log 6}}$ • divide to solve for "x".

Reorder: $\boxed{x = \frac{3\log 6 - 5\log 3}{\log 3 + 6\log 6}}$

w/calc. $\boxed{x \approx -.010}$

REVIEW

ex: Condense.

$$2\log(x+5)-\log(x+1)$$

$$\log(x+5)^2-\log(x+1)$$

$$\boxed{\log\left[\frac{(x+5)^2}{(x+1)}\right]}$$

Review

ex: Sketch.

$$y = \log_2(-x) + 5$$

$$\frac{-x}{-1} > \frac{0}{-1}$$

$x < 0$ domain

$x = 0$ VA

$y = \log_2(-x) + 5$

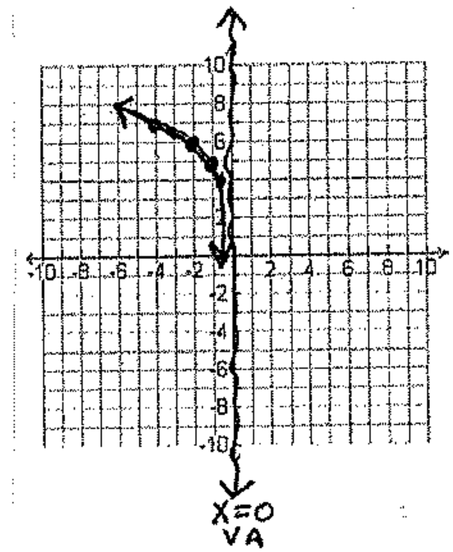
$y - 5 = \log_2(-x)$

$-1\left((2)^{(y-5)} = -x\right)$

$-1 \cdot (2)^{(y-5)} = x$

Reorder: $x = -1 \cdot (2)^{(y-5)}$ $(y-5)=0$

x	y
$-\frac{1}{2}$	4
-1	5 *
-2	6
-4	7



$$\boxed{\begin{array}{l} D: \{x \mid x < 0\} \\ R: \{y \mid y \in \mathbb{R}\} \end{array}}$$

Review

ex: Expand.

$$\log_5 \sqrt{\frac{x-2}{25y^3x^7}}$$

$$= \log_5 \left(\frac{x-2}{25y^3x^7}\right)^{1/2}$$

$$= \frac{1}{2} \log_5 \left(\frac{x-2}{25y^3x^7}\right)$$

$$= \frac{1}{2}\left[\log_5(x-2) - \log_5 25 - \log_5 y^3 - \log_5 x^7\right]$$

$$= \frac{1}{2}\left[\log_5(x-2) - 2 - 3\log_5 y - 7\log_5 x\right]$$

$$= \boxed{\frac{1}{2}\log_5(x-2) - 1 - \frac{3}{2}\log_5 y - \frac{7}{2}\log_5 x}$$