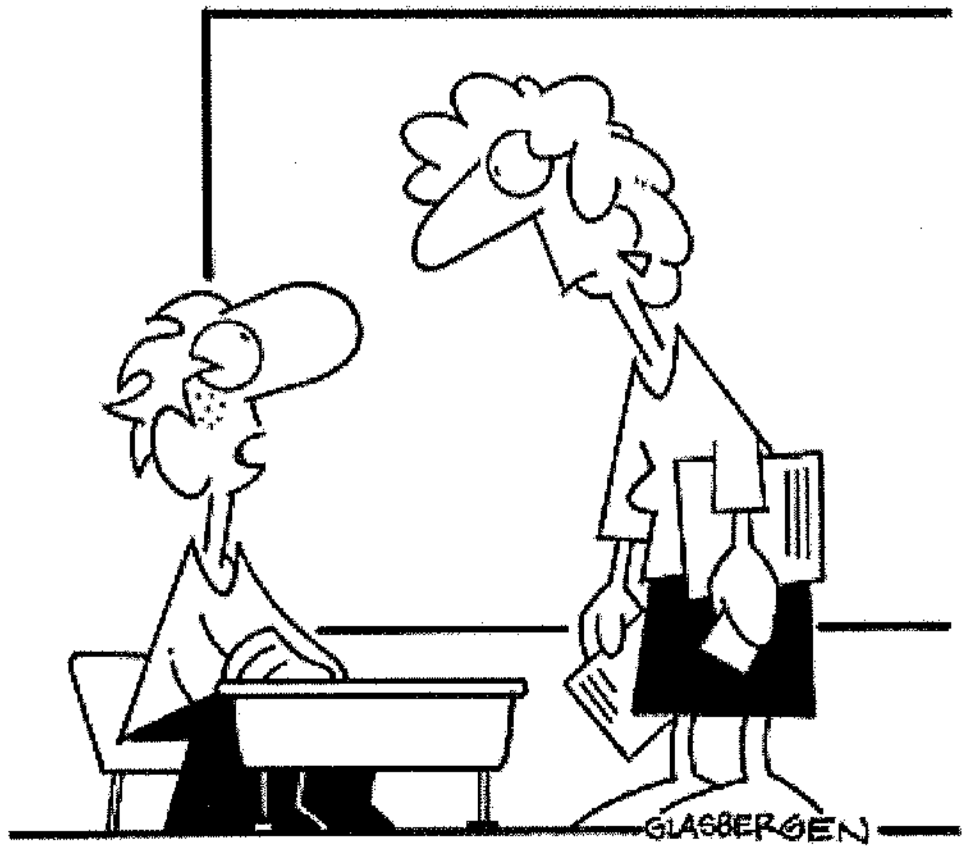
A2: Finding Inverses of Exponential and Logarithmic Functions, Solving Exponential and Logarithmic Equations - Mixed Practice

Notes



"If I work hard, I'll get good grades. If I get good grades, I'll go to a top college. If I go to a top college, I'll get a great job. If I get a great job, I'll make a lot of money. If I make a lot of money, everyone will hate me. That's why I didn't do my homework."

ex: Find the inverse.

a) $f(x) = 2^{x+1} - 3$

$$y = 2^{x+1} - 3$$

Switch x & y

$$x = 2^{y+1} - 3$$

Solve for y

$$x+3 = 2^{y+1}$$

$$\log(x+3) = \log 2^{(y+1)}$$

$$\frac{\log(x+3)}{\log 2} = \frac{(y+1)\cancel{\log 2}}{\cancel{\log 2}}$$

Reorder: $y+1 = \dfrac{\log(x+3)}{\log 2}$

Use change of base rule to combine.

$$y+1 = \log_2(x+3)$$

$$y = \log_2(x+3) - 1$$

Inverse notation

$$\boxed{f^{-1}(x) = \log_2(x+3) - 1}$$

or

$$\boxed{f^{-1}(x) = -1 + \log_2(x+3)}$$

ex: Find the inverse.

b) $f(x) = \log_5(x-5) + 2$

Switch x & y

$y = \log_5(x-5) + 2$

Isolate the "log"

$x = \log_5(y-5) + 2$

$(x-2) = \log_5(y-5)$

Change to exponential form

$5^{(x-2)} = y-5$

$5^{(x-2)} + 5 = y$

Reorder:

$y = 5^{(x-2)} + 5$

Inverse notation

$$\boxed{f^{-1}(x) = 5^{(x-2)} + 5}$$

ex: Find the inverse.

$\boxed{\text{Recall: } \ln \rightarrow \log_e}$

c) $h(x) = \ln x - 1$

$y = \ln x - 1$

Switch x & y

$x = \ln y - 1$

Isolate "log"

$x + 1 = \ln y$

change to exponential form

$(x+1) = \log_e y$

$e^{(x+1)} = y$

Reorder

$y = e^{(x+1)}$

inverse notation

$\boxed{h^{-1}(x) = e^{(x+1)}}$

ex: Find the inverse.

d) $g(x) = -5^{x-6}$

$$y = -5^{x-6}$$

$$-1\left(x = -5^{y-6}\right)$$

$$-x = 5^{y-6}$$

$$\log(-x) = \log 5^{(y-6)}$$

$$\frac{\log(-x)}{\log 5} = \frac{(y-6)\cancel{\log 5}}{\cancel{\log 5}}$$

Reorder

$$y - 6 = \frac{\log(-x)}{\log 5}$$

use change of base rule to combine

$$y - 6 = \log_5(-x)$$

$$y = \log_5(-x) + 6$$

$$\boxed{g^{-1}(x) = \log_5(-x) + 6}$$

or

$$\boxed{g^{-1}(x) = 6 + \log_5(-x)}$$

ex: Solve. Round to 3 decimal places when necessary and check for extraneous solutions.

a) $5^{x+3} + 1 = 17$

$\quad\;\; -1 \quad -1$

$$5^{(x+3)} = 16$$

$$\log 5^{(x+3)} = \log 16$$

$$\frac{(x+3)\cancel{\log 5}}{\cancel{\log 5}} = \frac{\log 16}{\log 5}$$

$$x+3 = \frac{\log 16}{\log 5}$$

Then w/ calc.

$$\boxed{x \approx -1.277}$$

w/ change of base

$$\boxed{x = -3 + \frac{\log 16}{\log 5}} \quad \underline{\underline{or}} \quad \boxed{x = -3 + \log_5 16}$$

ex: Solve. Round to 3 decimal places when necessary and check for extraneous solutions.

b) $\underbrace{\log_7(x-2) + \log_7(x+3)}$ $= \log_7 14$

• condense

$$\cancel{\log_7}\left[\underset{\text{Foil}}{(x-2)(x+3)}\right] = \cancel{\log_7} 14$$

$$x^2 + x - 6 = 14$$
$$\quad -14 \quad \cancel{-14}$$

$$x^2 + x - 20 = 0$$

$$(x+5)(x-4) = 0$$

$$\cancel{x = -5} \quad \Big| \quad \boxed{x = 4}$$

extraneous ✓

check each

ex: Solve. Round to 3 decimal places when necessary and check for extraneous solutions.

c) $\log_3(7x+3) = \log_3(5x+9)$

$$7x + \cancel{3} = 5\cancel{x} + 9$$
$$-5x - \cancel{3} \quad -\cancel{5}x - 3$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{6}{2}$$

$$\boxed{x = 3} \checkmark$$

Check

ex: Solve. Round to 3 decimal places when necessary and check for extraneous solutions.

d) $\left(\frac{1}{3}\right)^{x-1} = 6$

can't get the same base.

$$\log\left(\frac{1}{3}\right)^{(x-1)} = \log 6$$

$$\frac{(x-1)\cancel{\log(1/3)}}{\cancel{\log(1/3)}} = \frac{\log 6}{\log(1/3)}$$

$$x-1 = \frac{\log 6}{\log(1/3)}$$

$$\boxed{x = 1 + \frac{\log 6}{\log(1/3)}} \text{ or } \boxed{x = 1 + \log_{\frac{1}{3}} 6}$$

w/ calc.

$$\boxed{x \approx -0.631}$$

ex: Solve. Round to 3 decimal places when necessary and check for extraneous solutions.

e) $\log(5x - 11) = 2$

$$\log_{10}(5x-11) = 2$$

change to exponential form

$$10^2 = 5x - 11$$

$$100 = 5x - \cancel{11}$$

$$+11 \qquad \cancel{+11}$$

$$\frac{111}{5} = \frac{\cancel{5}x}{\cancel{5}}$$

$$\boxed{x = \frac{111}{5}} \checkmark$$

Check:

$$\log(5x-11)$$

$$\log\left[\cancel{5}\left(\frac{111}{\cancel{5}}\right)-11\right]$$

$$\log(111-11)$$

$$\log_{10}(100)$$

$$2 = 2$$

ex: Solve. Round to 3 decimal places when necessary and check for extraneous solutions.

f) $\left(\frac{1}{3}\right)^{x-1} = 27^{x+9}$ ← can get the same base ... so NO LOGS needed.

$$(\cancel{3}^{-1})^{(x-1)} = (\cancel{3}^{3})^{(x+9)}$$

$$-1(x-1) = 3(x+9)$$

$$\cancel{-x} + 1 = 3x + \cancel{27}$$
$$\cancel{+x} - 27 \qquad +x \quad \cancel{-27}$$

$$\frac{-26}{4} = \frac{\cancel{4}x}{\cancel{4}}$$

$$x = \frac{-26}{4} \div 2 \qquad \boxed{x = -\frac{13}{2}}$$

ex: Solve. Round to 3 decimal places when necessary and check for extraneous solutions.

g) $5^{2x+1} = 8$ ← can't get same base, so need logs

$$\log 5^{(2x+1)} = \log 8$$

$$\frac{(2x+1)\cancel{\log 5}}{\cancel{\log 5}} = \frac{\log 8}{\log 5}$$

$$2x+1 = \frac{\log 8}{\log 5}$$

$$\frac{1}{\cancel{2}} \cdot \left[\cancel{2}x\right] = \frac{1}{2}\left[-1 + \frac{\log 8}{\log 5}\right]$$

$$\boxed{x = \frac{1}{2}\left[-1 + \frac{\log 8}{\log 5}\right]} \quad \text{or} \quad \boxed{x = \frac{1}{2}\left[-1 + \log_5 8\right]}$$

w/ calc.

$$\boxed{x \approx 0.146}$$

ex: Solve. Round to 3 decimal places when necessary and check for extraneous solutions.

h) $\log_4(2x+1) = \underbrace{\log_4(x+2) - \log_4 3}_{\text{• condense}}$

$$\cancel{\log_4}(2x+1) = \cancel{\log_4}\left[\frac{x+2}{3}\right]$$

$$\frac{2x+1}{1} = \frac{x+2}{3}$$ cross multiply

$$3(2x+1) = x+2$$

$$6x + \cancel{3} = \cancel{x} + 2$$
$$-x \ \cancel{-3} \quad -\cancel{x} \ -3$$

$$5x = -1$$

$$✓ \boxed{x = \frac{-1}{5}}$$ check

ex: Solve. Round to 3 decimal places when necessary and check for extraneous solutions.

i) $3 - \ln(4x - 3) = 0$

$$3 = \ln(4x-3)$$ ← change to exponential form

$$3 = \log_e(4x-3)$$

$$e^3 = 4x - \cancel{3}$$
$$+3 \qquad \cancel{+3}$$

$$\frac{e^3+3}{4} = \frac{\cancel{4}x}{\cancel{4}}$$

$$\boxed{x = \frac{e^3+3}{4}}$$ check ✓

$$\boxed{x \approx 5.771}$$

Check: $\ln(4x-3)$

$\ln[\cancel{4}(\frac{e^3+3}{\cancel{4}})-3)$

$\ln(e^3+\cancel{3}-\cancel{3})$

$\ln(e^3)$ ✓

ex: Solve. Round to 3 decimal places when necessary and check for extraneous solutions.

j) $2\log_2(5x+7) - \cancel{1} = 9$

$\quad\quad\quad\quad\quad\quad\quad\quad \cancel{+1} \quad +1$

$$\frac{\cancel{2}\log_2(5x+7)}{\cancel{2}} = \frac{10}{2}$$

$$\log_2(5x+7) = 5$$

$$2^5 = 5x+7$$

$$32 = 5x + \cancel{7}$$
$$-7 \quad\quad\quad \cancel{-7}$$

$$\frac{25}{5} = \frac{\cancel{5}x}{\cancel{5}}$$

$$\boxed{x=5}$$

check ✓

ex: Solve. Round to 3 decimal places when necessary and check for extraneous solutions.

k) $2 \cdot 3^{2x} + 7 = 25$

$\quad\quad\quad\quad -7 \quad -7$

$$\frac{\cancel{2} \cdot 3^{2x}}{\cancel{2}} = \frac{18}{2}$$

$$3^{2x} = 9$$

Can get same base, so NO LOGS

$$\cancel{3}^{2x} = \cancel{3}^2$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{2}{2}$$

$$\boxed{x = 1}$$

ex: Solve. Round to 3 decimal places when necessary and check for extraneous solutions.

challenge ★ 1) $\underbrace{\log_5 x + \log_5 (x - 12)}_{\text{• condense}} = 3$

$\log_5 [x(x-12)] = 3$

$\log_5 (x^2 - 12x) = 3$

$5^3 = x^2 - 12x$

$125 = x^2 - 12x$

$0 = x^2 - 12x - 125$

does not factor
use QF

$a = 1$
$b = -12$
$c = -125$

$x = \dfrac{12 \pm \sqrt{144 - 4(1)(-125)}}{2(1)}$

$x = \dfrac{12 \pm \sqrt{644}}{2}$

$x = \dfrac{12 + \sqrt{644}}{2}$

$\boxed{x \approx 18.689}$

$\cancel{x = \dfrac{12 - \sqrt{644}}{2}}$

$\cancel{x \approx -6.689}$ extraneous