

Notes

Sketching Polynomial Functions

To sketch a polynomial function you will need the...

- a. x-intercepts (real zeros; determine whether the graph will ^{odd mult} cross or ^{even mult} bounce at each zero)
- b. y-intercept (plug in 0 for x)
- c. degree and maximum number of turning points
- d. end behavior
- e. table of values (for extra values... as needed)

plugin "0"
for x.

State the degree, y-intercept, and end behavior. Then match each polynomial to its graph.

1. $y = x^4 - x^2$
D: 4 even $\uparrow\uparrow \downarrow\downarrow$
 LC: 1 \oplus EB:

2. $y = x^3 - x$
D: 3 odd $\downarrow\uparrow \uparrow\downarrow$
 LC: 1 \oplus

3. $y = -x^4 + 1$
D: 4 even $\uparrow\uparrow \downarrow\downarrow$
 LC: -1 **B**

4. $y = -x^3 + x$
D: 3 odd $\downarrow \uparrow \downarrow$
 LC: -1

C
 $x \rightarrow -\infty, y \rightarrow \infty$
 $x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow -\infty$
 $x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow -\infty$
 $x \rightarrow \infty, y \rightarrow -\infty$

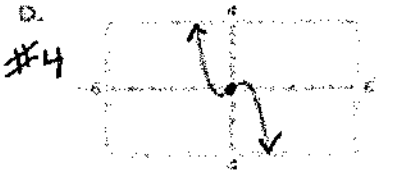
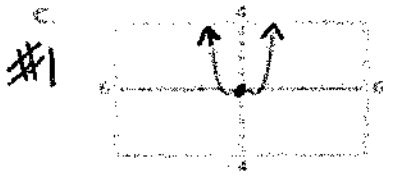
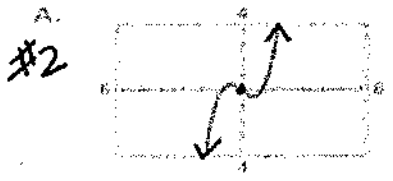
$x \rightarrow -\infty, y \rightarrow \infty$
 $x \rightarrow \infty, y \rightarrow -\infty$

y-int: (0,0)

y-int: (0,0) **A**

y-int: (0,1)

y-int: (0,0) **D**



$$\begin{array}{l}
 -x=0 \\
 x=0 \quad x+2=0 \\
 \quad \quad \quad x=-2 \\
 \quad \quad \quad \quad x-1=0 \\
 \quad \quad \quad \quad \quad x=1
 \end{array}$$

1. Sketch: $f(x) = -x(x+2)(x-1)$

a. X-int.

Z	M	C/B?
0	1	C
-2	1	C
1	1	C

b. Y-int.

$$f(0) = -0(0+2)(0-1) = 0$$

Y-int: (0,0)

c. D: 3

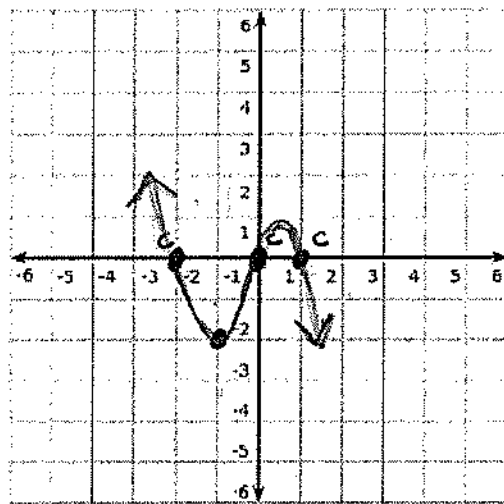
MTP: 2

d.

odd \downarrow or \uparrow

LC: -1

EB: $\begin{array}{l} X \rightarrow -\infty, y \rightarrow \infty \\ X \rightarrow \infty, y \rightarrow -\infty \end{array}$



e.

X	Y	$f(-1) = -(-1)(-1+2)(-1-1)$
-1	-2	$= 1(1)(-2)$
		$= -2$

1st factor: $x^4 - 5x^2 + 4 = 0$

$$(x^2 - 1)(x^2 - 4) = 0$$

$\begin{matrix} \text{DOS} & \text{DOS} \end{matrix}$

 $\begin{matrix} x^2 & x^2 \\ -1 & -4 \end{matrix}$

$$(x+1)(x-1)(x+2)(x-2) = 0$$

Then solve: $x = -1$ $x = 1$ $x = -2$ $x = 2$

2. Sketch: $g(x) = x^4 - 5x^2 + 4$

a. X-int:

Z	M	B/C
-1	1	C
1	1	C
-2	1	C
2	1	C

b. y-int:

$(0, 4)$

c. $D: 4$ even

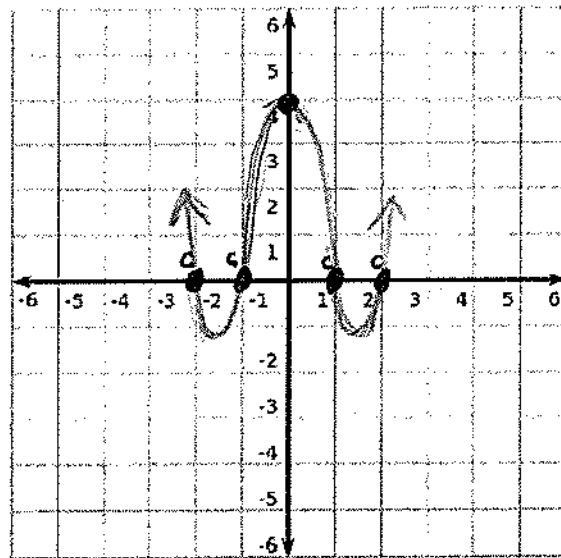
MTP: 3

d. EB?

$\uparrow \uparrow$ or $\downarrow \downarrow$

LC: \oplus

$x \rightarrow -\infty, y \rightarrow \infty$
$x \rightarrow \infty, y \rightarrow \infty$



e.

x | y

no extra pts needed here.

$x-1=0$
 $x=1$

$x+2=0$
 $x=-2$
 mult: 2

3. Sketch: $h(x) = \frac{1}{2}(x-1)(x+2)^2$

a. X-int:

x	M	B/C?
1	1	C
-2	2	B

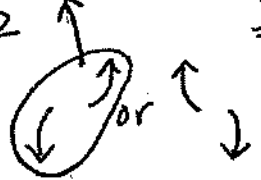
b. y-int:
 $(0, -2)$

$$\begin{aligned}
 h(0) &= \frac{1}{2}(0-1)(0+2)^2 \\
 &= \frac{1}{2}(-1)(2)^2 \\
 &= \frac{1}{2}(-1)(4) \\
 &= \frac{1}{2}(-4) \\
 &= -2
 \end{aligned}$$

c. $D: 3$ odd

MTP: 2

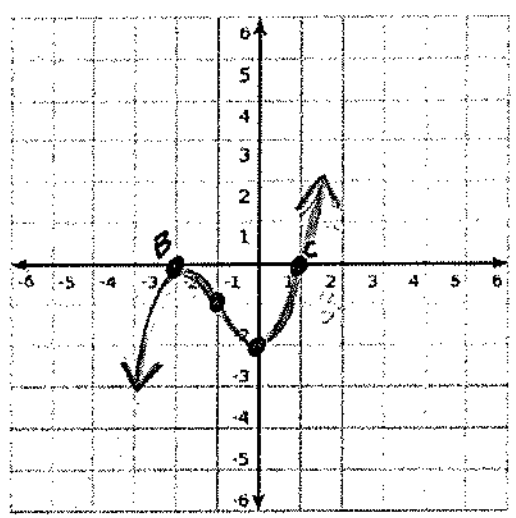
d. EB?



LC: $\frac{1}{2}$
 \oplus

$x \rightarrow -\infty, y \rightarrow -\infty$

$x \rightarrow \infty, y \rightarrow \infty$



e.

x	y
-1	-1

$$\begin{aligned}
 h(-1) &= \frac{1}{2}(-1-1)(-1+2)^2 \\
 &= \frac{1}{2}(-2)(1)^2 \\
 &= \frac{1}{2}(-2)(1) \\
 &= -1
 \end{aligned}$$

$$x^3 - x^2 - 4x + 4 = 0$$

$$x^2(x-1) - 4(x-1) = 0$$

$$(x-1)(x^2 - 4) = 0$$

$$(x-1)(x+2)(x-2) = 0$$

$$x=1 \quad x=-2 \quad x=2$$

4. Sketch: $m(x) = x^3 - x^2 - 4x + 4$

a. x-int:

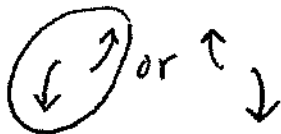
x	M	$B/C?$
1	1	C
-2	1	C
2	1	C

b. y-int: $(0, 4)$

c. $D: 3$ odd

MTP: 2

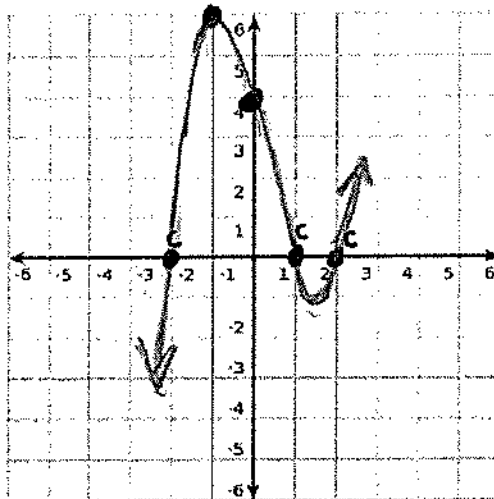
d. EB:



LC: 1

$$x \rightarrow -\infty, y \rightarrow -\infty$$

$$x \rightarrow \infty, y \rightarrow \infty$$



e.

x	y
-1	6

$$m(-1) = (-1)^3 - (-1)^2 - 4(-1) + 4$$

$$= -1 - 1 + 4 + 4$$

$$= -2 + 8$$

$$= 6$$

⊗ or plug into the factored form:

$$m(x) = (x-1)(x+2)(x-2)$$

$$m(-1) = (-1-1)(-1+2)(-1-2)$$

$$= (-2)(1)(-3)$$

$$= 6$$

$$4x^2 - 4x^3 = 0$$

gcf:

$$4x^2(1-x) = 0$$

$$x=0 \quad 1-x=0 \\ 1=x$$

5. Sketch: $p(x) = 4x^2 - 4x^3$

a. x-int:

z	M	B/C?
0	2	B
1	1	C

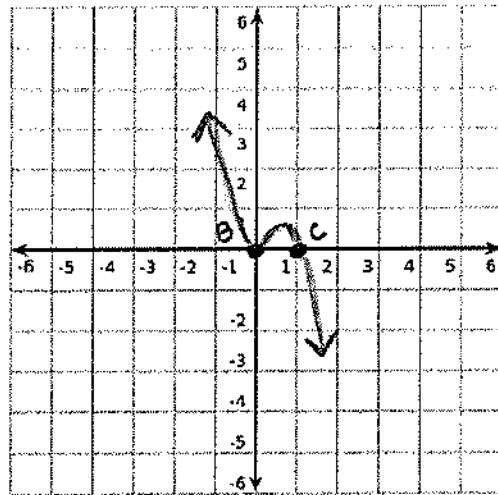
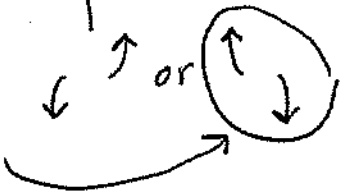
b. y-int: $(0,0)$

c. $D: 3$ odd

MTP: 2

d. EB:

LC: -4



e. $\frac{x}{y}$

no extra points needed here.

$$-x^4 + x^2 = 0$$

$$\text{gcf: } -x^2(x^2 - 1) = 0$$

$$-x^2(x+1)(x-1) = 0$$

$$x=0 \quad x=-1 \quad x=1$$

6. Sketch: $y = -x^4 + x^2$

a. x-int.

Z	M	B/C?
0	2	B
-1	1	C
1	1	C

b. y-int: $(0,0)$

c. $D: 4$ even

MTP: 3

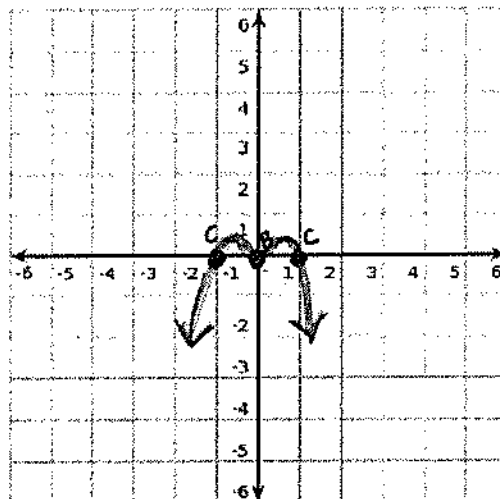
d. EB:

LC: -1

$\uparrow \uparrow$ or $\downarrow \downarrow$

$$x \rightarrow -\infty, y \rightarrow -\infty$$

$$x \rightarrow \infty, y \rightarrow -\infty$$



e.

X | Y

no extra points needed here.