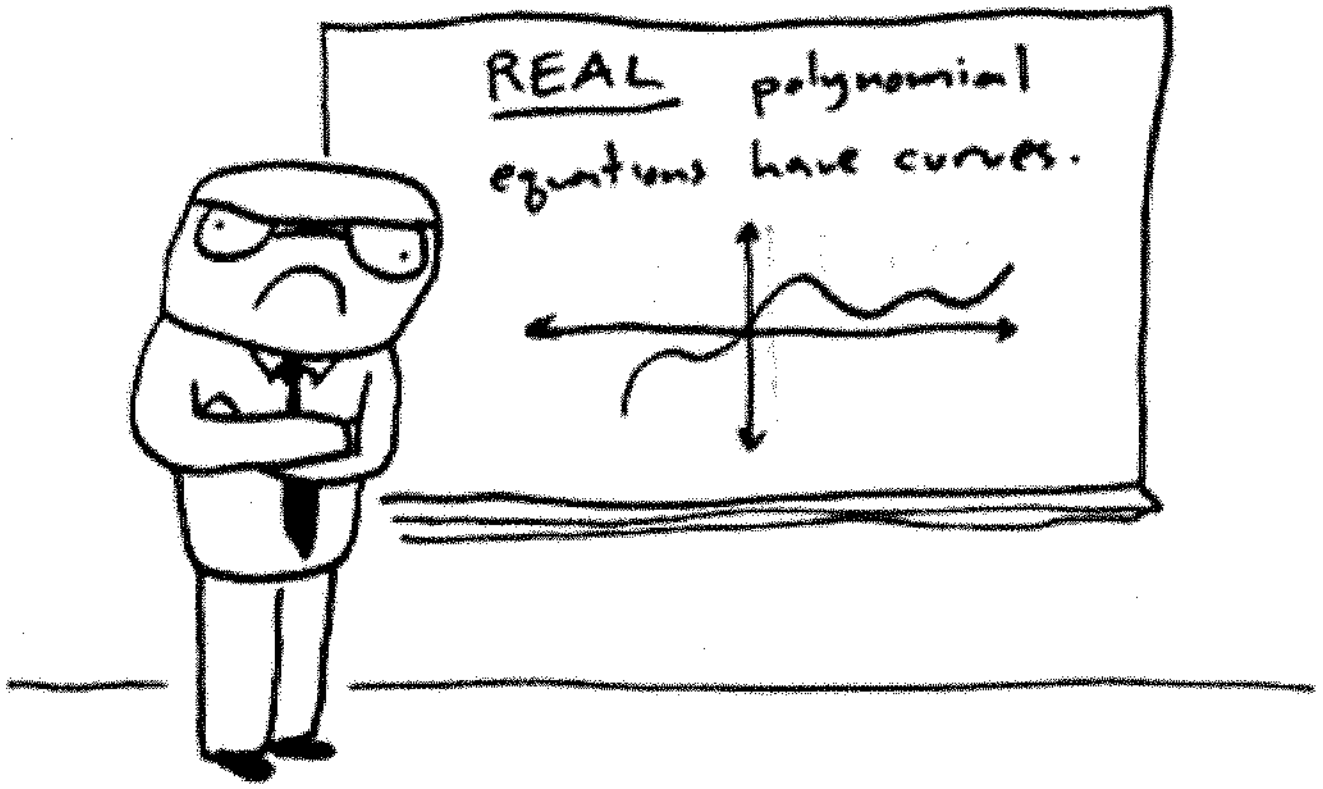
Notes

Apply the Remainder and Factor Theorems
Find Rational Zeros
Introduction to the Rational Zero Theorem



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The Factor Theorem

A polynomial $g(x)$ is a factor of $f(x)$ if...

1. $\frac{f(x)}{g(x)}$ has a remainder of 0.

When using Synthetic or long division.

2. k is a zero of $g(x)$ and $f(k) = 0$.

ex: Is $g(x)$ a factor of $f(x)$?

a) $g(x) = x - 5$, $f(x) = x^3 - 7x^2 + 7x + 15$

no RT's

$x - 5 = 0$

$x = 5$ →

Use synthetic division

5	1	−7	+7	+15
	↓	+5	−10	−15
	1	−2	−3	0 ← R

$R = 0$ ∴

Yes, g(x) is a factor of f(x).

ex: Is $g(x)$ a factor of $f(x)$?

b) $g(x) = x + 7, \quad f(x) = x^2 - 9$ add MT's

$= x^2 + 0x - 9$

$x + 7 = 0$

$x = -7$

use synthetic division

$$\begin{array}{r|rrr} -7 & 1 & 0 & -9 \\ & \downarrow & -7 & +49 \\ \hline & 1 & -7 & +40 \leftarrow R \end{array}$$

$R \neq 0$ ∴

No, $g(x)$ is not a factor of $f(x)$.

ex: Factor $f(x)$ completely given one of its factors.

not solve

a) $f(x) = 15x^3 + x^2 - 22x - 8;$ $x+1$

D3

no mT's

$X+1=0$

$X=-1 \leftarrow$ Box

-1	15	$+1$	-22	-8
	↓	-15	$+14$	$+8$
D2 →	15	-14	-8	$0 \leftarrow R$

R=0 B/c $X+1$ is a factor of $f(x)$.

Given 2

$(X+1)(15X^2 - 14X - 8) = 0$

$(X+1)(5X+2)(3X-4) = 0$

-120

$4 \quad 30$

$5 \quad 24$

$15X + 6 \div 3 \quad 15X - 20 \div 5$

$\frac{5X}{+2} \quad \frac{3X}{-4}$

The Rational Zero Theorem

If $f(x)$ is a polynomial then every rational zero of $f(x)$ comes in the form of...

$$\frac{P}{q} : \frac{\text{factors of constant term}}{\text{factors of leading coefficient}}$$

$5x^3 - 2x^2 - 7x + 4$

↑ Leading coefficient q

↑ Constant P

$P = 4$

$q = 5$

ex: List the possible rational zeros.

a) $f(x) = 2x^3 - 7x^2 + 9$

q ↑ ↖ p

9: 1, 9, 3, 3

2: 1, 2

Factors of each:

p: $\pm 1, \pm 3, \pm 9$

q: $\pm 1, \pm 2$

$$\frac{p}{q} : \pm 1, \pm 3, \pm 9, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{9}{2}$$

possible rational zeros

ex: List the possible rational zeros.

b) $f(x) = 4x^4 - x^3 - 7x^2 + 4x - 2$

q ↑ p ↑

2: 1, 2 — 4: 1, 4, 2, 2

Factor of each:

p: $\pm 1, \pm 2$

q: $\pm 1, \pm 2, \pm 4$

$\frac{p}{q}$: $\pm 1, \pm 2, \pm \frac{1}{2}, \cancel{\pm \frac{2}{2}}, \pm \frac{1}{4}, \cancel{\pm \frac{2}{4}}$

same

same

$\frac{p}{q}$: $\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{4}$

possible rational zeros