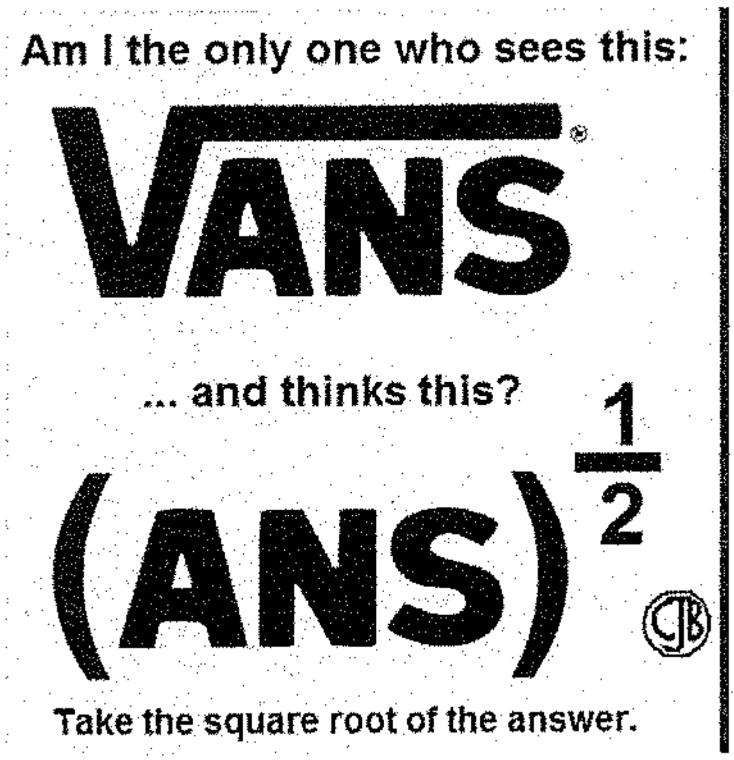
Notes

A2: Rational Exponents



Rational Exponents

Fractions as exponents

$$\boxed{x^{\frac{e}{i}} = \left(\sqrt[i]{x}\right)^{e}} \text{ or } \sqrt[i]{x^{e}}$$

Where:

- e is called the exponent (numerator)
- i is called the index (denominator)

only 2

ex: Rewrite in radical form.

a) $4^{3/5}$ $= \boxed{\left(\sqrt[5]{4}\right)^3}$

index = 5

b) $27^{4/3}$ $= \boxed{\left(\sqrt[3]{27}\right)^4}$

index = 3

c) $9^{5/2}$ $= \boxed{\left(\sqrt{9}\right)^5}$

index = 2

only 2

ex: Rewrite in radical form.

* d) $-4^{1/2} = -1 \cdot 4^{1/2} = -1 \cdot \sqrt{4} = \boxed{-\sqrt{4}}$

no parenthesis

index = 2

e) $(-3)^{5/3} = \boxed{\left(\sqrt[3]{-3}\right)^5}$

index = 3

f) $2 \cdot 7^{3/4} = 2 \cdot \left(\sqrt[4]{7}\right)^3$

index = 4

$= \boxed{2\left(\sqrt[4]{7}\right)^3}$

ex: Rewrite in exponential form.

a) $\sqrt[7]{21^1} = \boxed{21^{1/7}}$

index = 7

b) $\sqrt{8^3} = \boxed{8^{3/2}}$

index = 2

* c) $\sqrt[3]{-9} = -9^{\frac{1}{3}}$ ~~or~~ $\boxed{(-9)^{1/3}}$

not this one

needs parenthesis here

index = 3

ex: Evaluate. If no real value exists, write "nonreal."

a) $9^{3/2} = \left(\sqrt{9}\right)^3 = (3)^3 = \boxed{27}$

index = 2

b) $81^{3/4} = \left(\sqrt[4]{81}\right)^3 = (3)^3 = \boxed{27}$

$81 = 3^4$

index = 4

c) $4^{5/2} = \left(\sqrt{4}\right)^5 = (2)^5 = \boxed{32}$

index = 2

ex: Evaluate. If no real value exists, write "nonreal."

d) $8^{4/3}$ $= (\sqrt[3]{8})^4 = (2)^4 = \boxed{16}$

index = 3

e) $125^{2/3}$ $= (\sqrt[3]{125})^2 = (5)^2 = \boxed{25}$

$125 = 5^3$

$$\begin{array}{r|l} 5 & 125 \\ 5 & 25 \\ & 5 \end{array}$$

index = 3

f) $32^{3/5}$ $= (\sqrt[5]{32})^3 = (2)^3 = \boxed{8}$

index = 5

ex: Evaluate. If no real value exists, write "nonreal."

g) $(-216)^{2/3}$ $= \left(\sqrt[3]{-216}\right)^2$

index = 3 $\left(-\sqrt[3]{216}\right)^2$

$(-1\cdot 2\cdot 3)^2 = (-6)^2 = \boxed{36}$

$2\,|\underline{216}$
$2\,|\underline{108}$
$2\,|\underline{54}$
$3\,|\underline{27}$
$3\,|\underline{9}$
3

or
$216 = 6^3$

h) $(-16)^{5/2}$ $= \left(\sqrt{-16}\right)^5$

index = 2 $= \boxed{\text{nonreal}}$ ← negative under even root (even index)

* i) $-4^{3/2}$

no parenthesis

$= -1 \cdot 4^{3/2}$

$= -1 \cdot \left(\sqrt{4}\right)^3$

$= -1 \cdot (2)^3$

$= -1 \cdot 8$

$= \boxed{-8}$

ex: Evaluate. If no real value exists, write "nonreal."

j) $2 \cdot 32^{3/5} = 2 \cdot \left(\sqrt[5]{32}\right)^3$ $\quad 32 = 2^5$

$= 2 \cdot (2)^3$

$= 2 \cdot 8$

$= \boxed{16}$

k) $\left(\frac{81}{16}\right)^{3/4} = \left(\sqrt[4]{\frac{81}{16}}\right)^3 = \left(\frac{\sqrt[4]{81}}{\sqrt[4]{16}}\right)^3 = \left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \boxed{\frac{27}{8}}$

$81 = 3^4$

$16 = 2^4$

* l) $16^{-3/4} = \frac{1}{16^{3/4}} = \frac{1}{(\sqrt[4]{16})^3} = \frac{1}{(2)^3} = \boxed{\frac{1}{8}}$

remove the negative on the exponent

ex: Evaluate. If no real value exists, write "nonreal."

m) $9^{-5/2} = \frac{1}{9^{5/2}} = \frac{1}{(\sqrt{9})^5} = \frac{1}{(3)^5} = \boxed{\frac{1}{243}}$

remove the negative

remove

n) $-8^{-2/3} = -1 \cdot 8^{-2/3} = -1 \cdot \frac{1}{8^{2/3}} = -1 \cdot \frac{1}{(\sqrt[3]{8})^2}$

no parenthesis

$= -1 \cdot \frac{1}{(2)^2}$

$= -1 \cdot \frac{1}{4} = \boxed{-\frac{1}{4}}$

* Flip the fraction & make the exponent positive

* o) $\left(\frac{81}{4}\right)^{-3/2} = \left(\frac{4}{81}\right)^{3/2} = \left(\sqrt{\frac{4}{81}}\right)^3 = \left(\frac{\sqrt{4}}{\sqrt{81}}\right)^3 = \left(\frac{2}{9}\right)^3 = \frac{2^3}{9^3} = \boxed{\frac{8}{729}}$

* p) $\left(\frac{27}{8}\right)^{-4/3} = \left(\frac{8}{27}\right)^{4/3} = \left(\sqrt[3]{\frac{8}{27}}\right)^4 = \left(\frac{\sqrt[3]{8}}{\sqrt[3]{27}}\right)^4 = \left(\frac{2}{3}\right)^4$

$= \frac{2^4}{3^4}$

$= \boxed{\frac{16}{81}}$