Quadratic Word Problems

Notes

1) Read the problem carefully. Decide what unknown numbers are asked for and what facts are known. Making a sketch may help.

2) Choose a variable and use it with the given facts to represent the unknowns decribed in the problem.

3) Reread the problem and write an equation that represents relationships among the numbers in the problem.

4) Solve the equation and find the unknowns.

5) Check your results with the word of the problem.
State the answer. • Add units to the answer

Set up an equation then solve.

2) *The square of a number decreased by three times the number is 28. Find all possible values for the number.*

* Let n = number

mandatory

$$n^2 - 3n = 28$$
$$-28 \quad -28$$

$$n^2 - 3n - 28 = 0$$

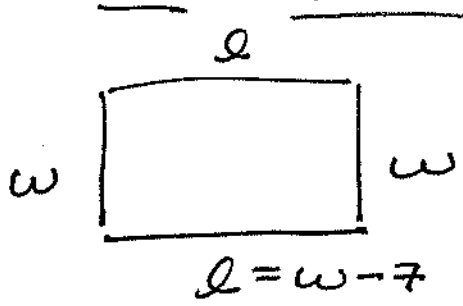
$$(n-7)(n+4) = 0$$

$n - 7 = 0$ | $n + 4 = 0$

$\boxed{n = 7}$ | $\boxed{n = -4}$

Set up an equation then solve.

3) The length of a rectangle is 7 less than its width. Find the perimeter if the area is 60 square units.



Let ℓ = length

w = width

A = area

$\ell = w - 7$

$A = 60 \text{ units}^2$

$A = \ell \cdot w$

$60 = (w-7) \cdot w$

$60 = w^2 - 7w$

$-60 \quad -60$

$0 = w^2 - 7w - 60$

$0 = (w+5)(w-12)$

-60

$w \quad w$

$+5 \quad -12$

$w + 5 = 0$ | $w - 12 = 0$

$w = -5$ | $w = 12$

extraneous

So... $w = 12$ units

and $\ell = w - 7$

$\ell = 12 - 7$

$\ell = 5$ units

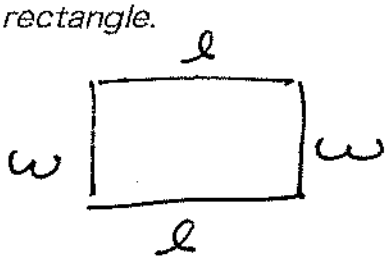
need perimeter (P): $P = 2\ell + 2w$

$P = 2(5) + 2(12) = 10 + 24$

Perimeter: **34 units**

Set up an equation then solve.

4) *The length of a rectangle is 6 cm more than three times its width. The area of the rectangle is 144 cm^2. Find the dimensions of the rectangle.*



Let l = length
w = width
A = Area

$l = 3w + 6$

$A = 144$

$A = l \cdot w$

$144 = (3w+6)w$

$144 = 3w^2 + 6w$

$-144 \quad -144$

$0 = 3w^2 + 6w - 144$

$0 = 3(w^2 + 2w - 48)$

$0 = 3(w-6)(w+8)$

$3 = 0$	$w - 6 = 0$	$w + 8 = 0$
no variable	$w = 6$	$w = -8$ extraneous

-48

$w \quad w$

$-6 \quad +8$

48
3 | 144
12↓
24
−24
0

So... $w = 6$

$l = 3w + 6$

$l = 3(6) + 6$

$= 18 + 6$

$= 24$

width = 6 cm

length = 24 cm

Dimensions: **6cm by 24cm**

Set up an equation then solve.

one right after the other

5) *Find two consecutive negative integers whose product is 90.*

multiplication

Let $1^{st}\#: n$

$2^{nd}\#: n+1$

$(1^{st}\#)\cdot(2^{nd}\#) = 90$

$n(n+1) = 90$

$n^2 + n = 90$

$n^2 + n - 90 = 0$

$(n+10)(n-9) = 0$

$n+10=0$ | $n-9=0$

$n=-10$ | $n=9$ (not a negative)

$1^{st}\#$

So . . .

$2^{nd}\# = n+1$

$= -10+1$

$2^{nd}\# = -9$

Answer : -10 and -9

Set up an equation then solve.

6) *Two consecutive positive odd integers have a product of 99. Find the numbers.*

Let $1^{st}\# = n$

$2^{nd}\# = n+2$ (odd)

$(1^{st}\#)\cdot(2^{nd}\#) = 99$

$n(n+2) = 99$

$n^2+2n = 99$

$n^2+2n-99 = 0$

$(n-9)(n+11) = 0$

$n-9=0$ | $n+11=0$

$n=9$ | $n=-11$ must be positive

n = $1^{st}\#$

So... $2^{nd}\# = n+2$

$= 9+2$

$= 11$

Answer: 9 and 11

Set up an equation then solve.

7) The product of two consecutive even integers is 48. What are the integers?

Let $1^{st}\# = n$

$2^{nd}\# = n+2$ (even)

$n(n+2) = 48$

$n^2+2n = 48$

$n^2+2n-48 = 0$

$(n+8)(n-6) = 0$

$n+8=0$ | $n-6=0$

$n=-8$ | $n=6$

* either could work

So . . . $2^{nd}\# = n+2$

If $n=-8$

$n+2 = -6$

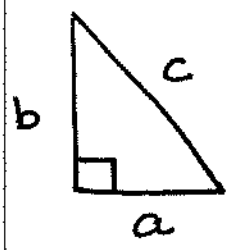
If $n=6$

$n+2 = 8$

Answer : **-8 and -6** or **6 and 8**

Set up an equation then solve.

8) A right triangle has sides that are consecutive even integers. Find the numbers.



Let 1st side = n

2nd side = $n+2$

3rd side = $n+2+2 = n+4$

Pythagorean Theorem:

$a^2+b^2=c^2$

$n^2+(n+2)^2=(n+4)^2$

$n^2+(n+2)(n+2)=(n+4)(n+4)$

Foil Foil

$n^2+n^2+2n+2n+4 = n^2+4n+4n+16$

$2n^2+4n+4 = n^2+8n+16$

$-n^2-8n-16 \quad -n^2-8n-16$

$n^2-4n-12=0$

$(n-6)(n+2)=0$

$n=6$ | ~~$n=-2$~~ extraneous

↑ 1st side

So... 2nd side $= n+2$

$= 6+2$

$= 8$

3rd side $= n+4$

$= 6+4$

$= 10$

Answer: 6, 8, 10

Set up an equation then solve.

9) The formula for throwing a baseball in the air is represented by $h(t) = -16t^2 + 12t + 40$, where h(t) is the height of the ball (in feet) and t is measured in seconds.

t → time

a) What is the height at t = 1 second?

$$h(1) = -16(1)^2 + 12(1) + 40$$
$$= -16(1) + 12 + 40$$
$$= -16 + 12 + 40$$
$$= -4 + 40$$
$$= \boxed{36 \text{ feet}}$$

add units

Set up an equation then solve.

9) The formula for throwing a baseball in the air is represented by $h(t) = -16t^2 + 12t + 40$, where h(t) is the height of the ball (in feet) and t is measured in seconds.

b) After how many seconds will the ball hit the ground?

$h(t) = 0$ (not $t = 0$)

$$0 = \frac{-16t^2}{-4} + \frac{12t}{-4} + \frac{40}{-4}$$

$$0 = -4(4t^2 - 3t - 10)$$

$$0 = -4(4t+5)(t-2)$$

$-4 = 0$ | $4t + 5 = 0$ | $t - 2 = 0$

$4t = -5$ | $t = 2$

$t = \frac{-5}{4}$

extraneous

-40

$4t$ $+5$

$4t$ -8

$\div 4$

$1t$ -2

2 seconds

Set up an equation then solve.

10) An object is launched up into the air and is modeled by $h(t) = -16t^2 + 64t + 6$ where h(t) is measured in feet and t is measured in seconds.

$a = -16 \quad b = 64 \quad c = 6$

a) At what time is the object at its maximum height? (X, Y)

input (t)

vertex (X, f(x))

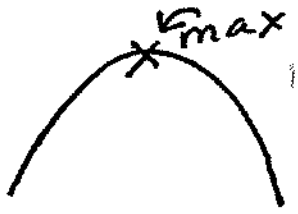
(t, h(t))

$$X = t = \frac{-b}{2a}$$

$$= \frac{-(64)}{2(-16)}$$

$$= \frac{-64}{-32}$$

$$t = 2$$



2 seconds

Set up an equation then solve.

10) An object is launched up into the air and is modeled by $h(t) = -16t^2 + 64t + 6$ where h(t) is measured in feet and t is measured in seconds.

b) What is the maximum height?

$t = 2$ vertex

$$h(2) = -16(2)^2 + 64(2) + 6$$
$$= -16(4) + 128 + 6$$
$$= -64 + 128 + 6$$
$$= 64 + 6$$
$$= 70$$

70 feet

Set up an equation then solve.

10) An object is launched up into the air and is modeled by $h(t) = -16t^2 + 64t + 6$ where h(t) is measured in feet and t is measured in seconds.

c) Find the time(s) when the object is at a height of 6 feet.

$6 = -16t^2 + 64t + 6$

$-6 \qquad\qquad -6$

$0 = -16t^2 + 64t$

$0 = -16t(t-4)$

$\frac{-16t}{-16} = \frac{0}{-16} \quad | \quad t - 4 = 0$

$t = 0 \quad | \quad t = 4$

$16\overline{)64}$ → 4, -64, 0

at 0 seconds and 4 seconds