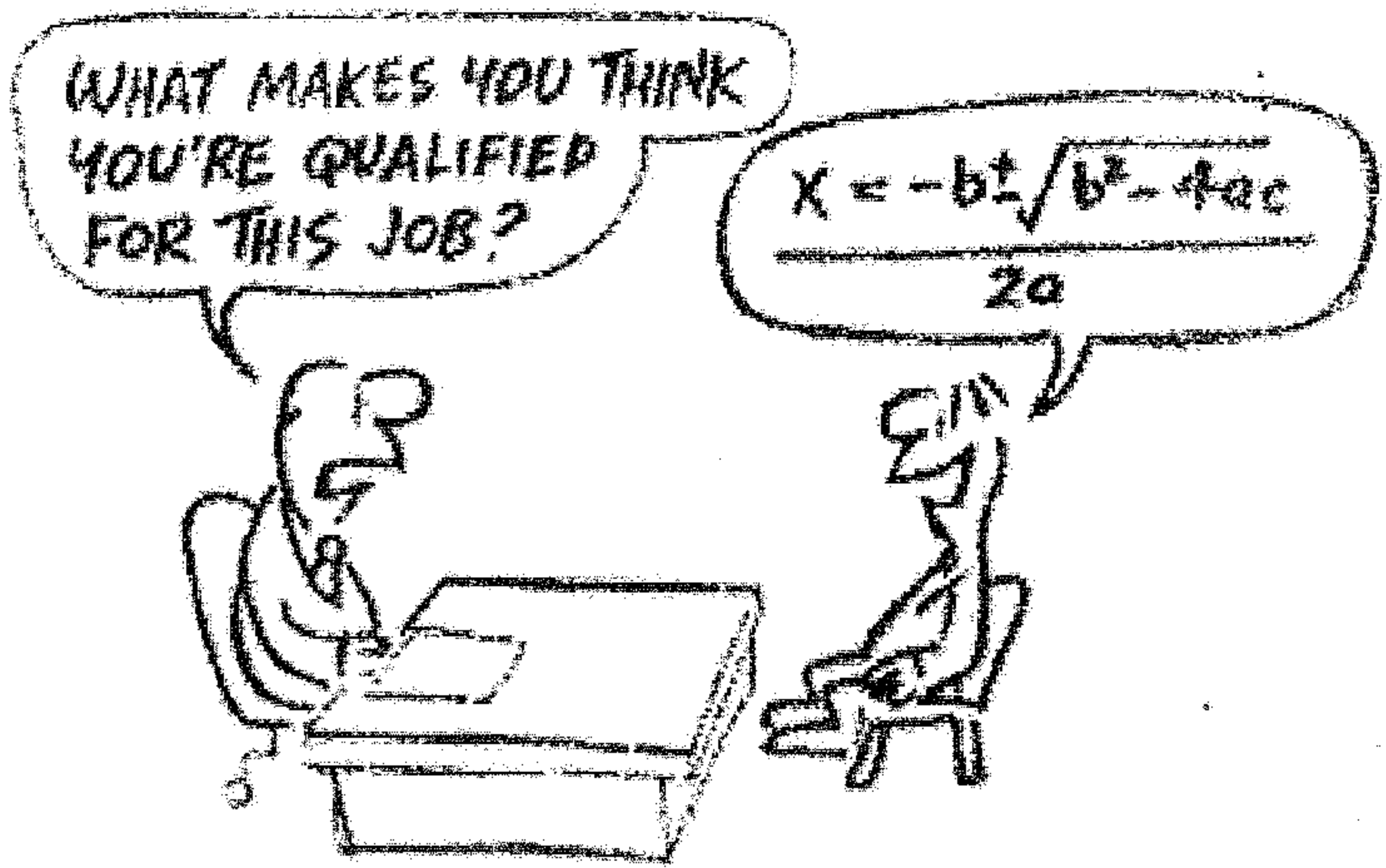
Quadratic Formula & The Discriminant

Notes



Solving Quadratics Using the Quadratic Formula

Let $a, b, c \in R$ such that $a \neq 0$. The solutions of the quadratic equation $ax^2 + bx + c = 0$ are:

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
(QF)

*Use the Quadratic Formula to solve a quadratic equation when...

- you can't factor (prime)
- you can't use square roots (SR)

ex: Solve.

① Standard form $= 0$

$$ax^2 + bx + c = 0$$

(*positive* "a")

a) $x^2 + 3x = 2$

$-2 \quad -2$

$$1x^2 + 3x - 2 = 0$$

$$\boxed{\begin{array}{l} a = 1 \\ b = 3 \\ c = -2 \end{array}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\underbrace{-4(1)(-2)}$$

$$-4(-2)$$

$$+8$$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 + 8}}{2}$$

$$\boxed{x = \frac{-3 \pm \sqrt{17}}{2}} \quad \text{or} \quad \boxed{x = \frac{-3}{2} \pm \frac{\sqrt{17}}{2}}$$

b) $-x^2+4x-5=0$

easier if $a>0$ (positive)

$$\frac{-x^2}{-1}+\frac{4x}{-1}\frac{-5}{-1}=\frac{0}{-1}$$

$$1x^2-4x+5=0$$

$$\boxed{\begin{aligned}a&=1\\ b&=-4\\ c&=5\end{aligned}}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(5)}}{2(1)}$$

$\underbrace{-4(1)}(5)$
$-4(5)$
-20

$$x=\frac{4\pm\sqrt{16-20}}{2}$$

$$x=\frac{4\pm\sqrt{-4}}{2}$$

$\sqrt{-4}$
$\sqrt{-1\cdot 4}$
$\sqrt{-1}\cdot\sqrt{4}$
$i\cdot 2$
$2i$

$$x=\frac{4\pm 2i}{2}$$

you may not reduce unless both terms reduce.

$$x=\frac{4}{2}\pm\frac{2i}{2}$$

$$\boxed{x=2\pm i}$$

c) $k^2 + 5k - 6 = 0$

Factoring

no gcf

-6

$-1 \quad +6$

$(K-1)(K+6) = 0$

$K-1=0$	$K+6=0$
$\boxed{K=1}$	$\boxed{K=-6}$

★ Much easier and quicker to <u>factor</u> than to use QF.

Quad. Formula (QF)

$-4(1)(-6)$

$-4(-6)$

$+24$

$\boxed{\begin{aligned} a &= 1 \\ b &= 5 \\ c &= -6 \end{aligned}}$

$$K = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(-6)}}{2(1)}$$

$$K = \frac{-5 \pm \sqrt{25 + 24}}{2}$$

$$K = \frac{-5 \pm \sqrt{49}}{2}$$ — perfect square

$$K = \frac{-5 \pm 7}{2}$$ DO NOT STOP HERE!

$K = \frac{-5+7}{2}$	$K = \frac{-5-7}{2}$
$K = \frac{2}{2}$	$K = \frac{-12}{2}$
$\boxed{K=1}$	$\boxed{K=-6}$

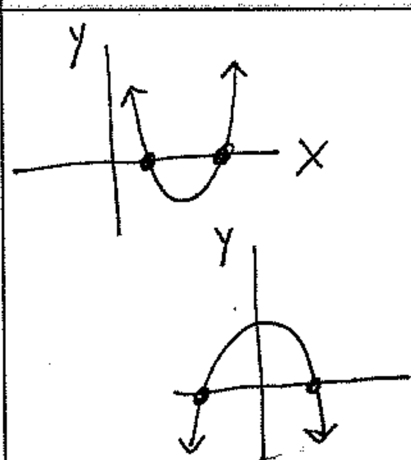
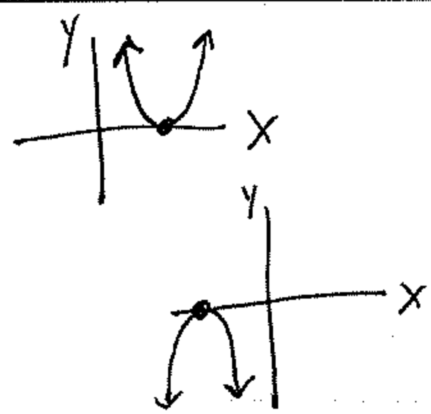
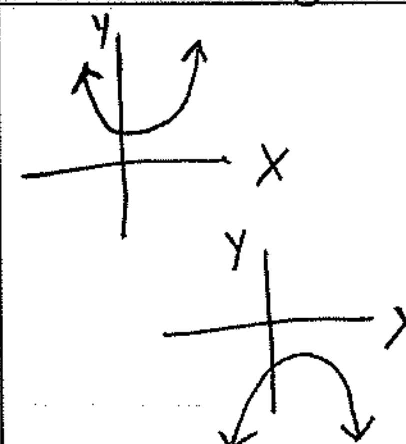
The Discriminant:

- In the quadratic formula, the expression b^2-4ac is called the discriminant.

  ↖ no square root on it.

- The discriminant is used to determine the number and type of solutions of a quadratic equation.

Using The Discriminant: b^2-4ac

Value of discriminant	+ (positive)	0 (zero)	− (negative)
Number of solutions	2 distinct	2 repeated (mult. 2)	2 distinct
Type of solutions	real	real	imaginary (i)
Graph of $y = ax^2 + bx + c$	 2 X-intercepts	 1 X-intercept	 0 x-intercepts (no x-int.)

D

ex: Find the discriminant and give the number and type of solutions of the equation.

$ax^2+bx+c=0$

Positive

set equal to zero

a) $x^2 - 8x + 13 = -4$

$\quad\quad\quad +4 \quad +4$

$1x^2 - 8x + 17 = 0$

$a = 1$ ← up

$b = -8$

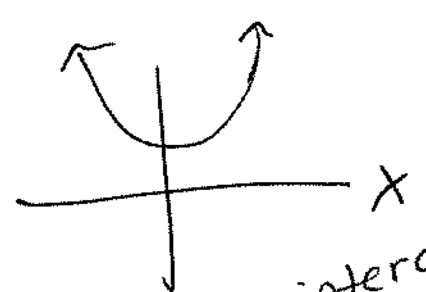
$c = 17$

$D = b^2 - 4ac$

$D = (-8)^2 - 4(1)(17)$

$= 64 - 68$

$D = \boxed{-4}$ ← negative



There would be no x-intercepts on the graph.

2 distinct imaginary solutions

b) $1x^2 - 8x + 16 = 0$

$a = 1$ ← up

$b = -8$

$c = 16$

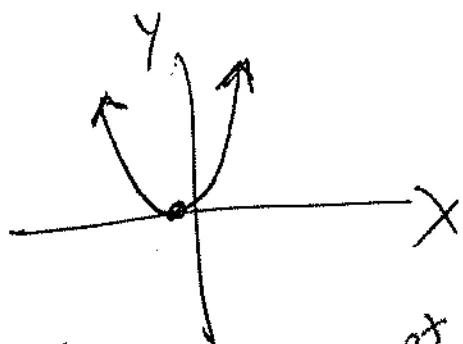
$D = b^2 - 4ac$

$D = (-8)^2 - 4(1)(16)$

$= 64 - 64$

$D = \boxed{0}$

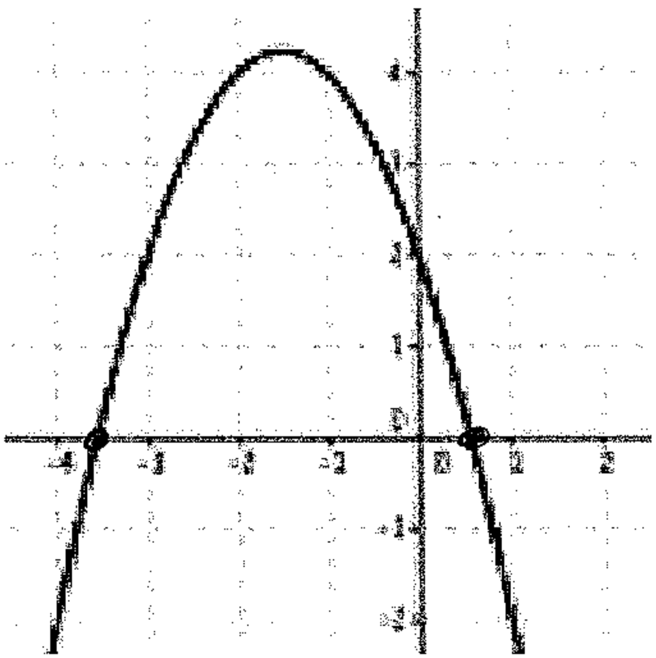
2 repeated **real** solutions



There would be 1 X-intercept on the graph.

ex: The graph of $y = ax^2 + bx + c$ or the solutions of $ax^2 + bx + c = 0$ are given. Determine if the discriminant is positive, negative, or zero. Explain your reasoning.

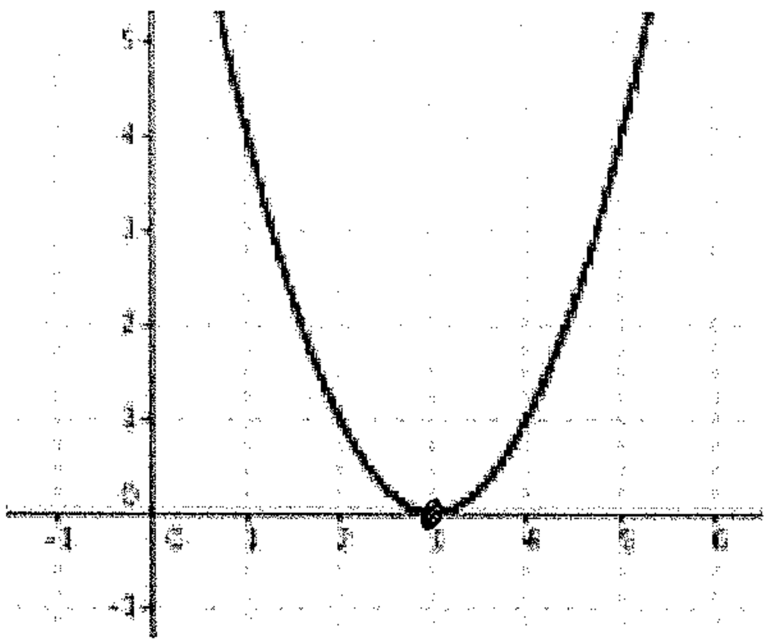
a)



2 x-intercepts

$\therefore D = \boxed{\text{Positive}}$

b)



1 x-intercept

$D = \boxed{0}$

c) $x = 2 \pm 3i$

imaginary #

$D = \boxed{\text{negative}}$

REVIEW

$$\boxed{\sqrt{-1} = \underline{\quad i \quad}}$$

ex: Simplify.

a) $\sqrt{-4}$

$\sqrt{-1 \cdot 4}$

$\sqrt{-1} \cdot \sqrt{4}$

$2i$

REVIEW

ex: Simplify.

b) $\sqrt{-108}$

$\sqrt{-1 \cdot 36 \cdot 3}$

$\sqrt{-1} \cdot \sqrt{36} \cdot \sqrt{3}$

$\boxed{6i\sqrt{3}}$

$\sqrt{-1 \cdot 9 \cdot 12}$

$3i\sqrt{12}$

$3i\sqrt{4 \cdot 3}$

$3i \cdot 2\sqrt{3}$

$6i\sqrt{3}$

$2 \,|\, 108$

$2 \,|\, 54$

$3 \,|\, 27$

$3 \,|\, 9$

3

$\underbrace{i \cdot 2 \cdot 3} \cdot \sqrt{3}$

$\boxed{6i\sqrt{3}}$

c) $-\sqrt{500}$

$-\sqrt{100 \cdot 5}$

$\boxed{-10\sqrt{5}}$

$5 \,|\, 500$

$10 \,|\, 100$

10

REVIEW

ex: Simplify.

d) $\sqrt{-\frac{4}{3}} = \sqrt{\frac{-4}{3}}$

$$\frac{\sqrt{-4}}{\sqrt{3}}$$

$\sqrt{-4}$

$\sqrt{-1} \cdot \sqrt{4}$

$i \cdot 2$

$2i$

$$\frac{2i}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2i\sqrt{3}}{\sqrt{9}}$$

$$\boxed{\frac{2i\sqrt{3}}{3}}$$

REVIEW

ex: Perform the indicated operation. Write your answer in standard form.

a) $3(4-3i)+5(2+i)$

$12 - 9i + 10 + 5i$

$= \boxed{22 - 4i}$

b) $(5-8i)^2$

$(5-8i)(5-8i)$

FOIL

$25 - 40i - 40i + \cancel{64i^2}$

-64

$= \boxed{-39 - 80i}$

$\boxed{i^2 = -1}$

$\begin{array}{r} 64 \\ -25 \\ \hline -39 \end{array}$

REVIEW

ex: Perform the indicated operation. Write your answer in standard form.

c) $$\frac{10}{(2+i)} \cdot \frac{(2-i)}{(2-i)} = \frac{20-10i}{4-\cancel{2i}+\cancel{2i}\underbrace{\cancel{-i^2}}_{+1}}$$

FOIL

Conjugate

$$= \frac{20-10i}{5}$$

$$= \frac{20}{5} - \frac{10i}{5}$$

$$= \boxed{4-2i}$$

REVIEW

ex: Perform the indicated operation. Write your answer in standard form.

d) $\frac{7}{i} \cdot \overset{1}{\frac{i}{i}} = \frac{7i}{i^2} = \frac{7i}{-1} = \boxed{-7i}$