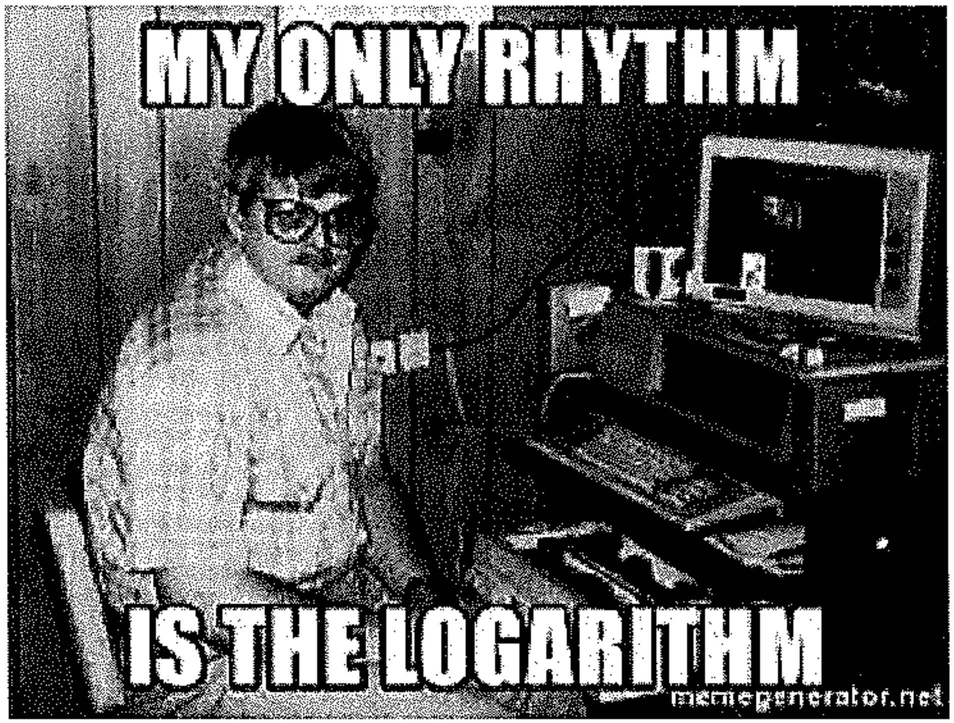
A2 - Properties of Logarithms Day 2

Notes



REVIEW: Logarithm Properties

Let b, m, and n be positive numbers such that $b \neq 1$.

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

ex: Expand.

a) $\log_3\left(\frac{abc}{9d}\right) = \log_3 a + \log_3 b + \log_3 c - \log_3 9 - \log_3 d$

$= \boxed{\log_3 a + \log_3 b + \log_3 c - 2 - \log_3 d}$

ex: Expand.

b) $\log\left(\dfrac{100a^2}{b^3c}\right) = \log 100 + \log a^2 - \log b^3 - \log c$

$= \boxed{2 + 2\log a - 3\log b - \log c}$

$\log_{10} 100 = \boxed{2}$

ex: Expand.

c) $\boxed{\log_3\left(a+b^2\right)}$

no rule
for add/sub
in the
argument
of a log

Does not
expand.

ex: Expand.

* d) $\log_4\left(\dfrac{a+b}{a^2-b^2}\right) = \log_4(a+b) - \log_4\underbrace{(a^2-b^2)}_{\text{DOS factor}}$

$$= \log_4(a+b) - {}^{*}\underbrace{\log_4[(a+b)\underset{\text{mult}}{\uparrow}(a-b)]}$$

$$= \log_4(a+b) - [\log_4(a+b) + \log_4(a-b)]$$

$$= \cancel{\log_4(a+b)} - \cancel{\log_4(a+b)} - \log_4(a-b)$$

$$= \boxed{-\log_4(a-b)}$$

ex: Expand.

e) $\log_2\left(a^2 - b^2\right) = \log_2[(a+b)\underset{\uparrow\,\text{mult.}}{}(a-b)]$

DOS

factor

$= \boxed{\log_2(a+b) + \log_2(a-b)}$

ex: Expand.

f) $\log_3(a-b)^7 = \boxed{7\log_3(a-b)}$

ex: Expand.

9) $$\ln\sqrt{\frac{y^3+z}{x^3(a+1)^5}} = \ln\left[\frac{y^3+z}{x^3(a+1)^5}\right]^{1/2}$$

$$= \frac{1}{2}\ln\left[\frac{y^3+z}{x^3(a+1)^5}\right]$$

$$= \frac{1}{2}\left[\ln(y^3+z) - \ln x^3 - \ln(a+1)^5\right]$$

$$= \frac{1}{2}\left[\ln(y^3+z) - 3\ln x - 5\ln(a+1)\right]$$

$$= \boxed{\frac{1}{2}\ln(y^3+z) - \frac{3}{2}\ln x - \frac{5}{2}\ln(a+1)}$$

ex: Expand.

h) $\log_2 \sqrt[3]{\dfrac{16a^5}{b^2+c^2}} = \log_2 \left(\dfrac{16a^5}{b^2+c^2}\right)^{1/3}$

$$= \frac{1}{3}\log_2\left(\frac{16a^5}{b^2+c^2}\right)$$

$$= \frac{1}{3}\left[\log_2 16 + \log_2 a^5 - \log_2(\underset{\text{prime}}{b^2+c^2})\right]$$

$$= \frac{1}{3}\left[4 + 5\log_2 a - \log_2(b^2+c^2)\right]$$

$$= \boxed{\frac{4}{3} + \frac{5}{3}\log_2 a - \frac{1}{3}\log_2(b^2+c^2)}$$

ex: Expand.

★ i) $\log_{32}\left(\frac{x^3 - y^3}{8}\right) = \log_{32}(x^3 - y^3) - \log_{32} 8$

$= \log_{32}(x^3 - y^3) - \frac{3}{5}$

$\log_{32} 8 =$

$\log_{2^5}(2^3) = \frac{3}{5}$

Diff. of cubes (SOAP)

$= \log_{32}[(x - y)(x^2 + xy + y^2)] - \frac{3}{5}$

S O AP

mult.

$= \boxed{\log_{32}(x - y) + \log_{32}(x^2 + xy + y^2) - \frac{3}{5}}$

prime trinomial

ex: Condense.

a) $2\log_5 a - 3\log_5 b + 4\log_5 (c+d)$

$= \log_5 a^2 - \log_5 b^3 + \log_5 (c+d)^4$

$= \boxed{\log_5 \left[\dfrac{a^2(c+d)^4}{b^3}\right]}$

ex: Condense.

b) $\frac{1}{2}\log x + \frac{3}{2}\log y - 10\log z$

$$= \log x^{1/2} + \log y^{3/2} - \log z^{10}$$

$$= \log\left[\frac{x^{1/2}y^{3/2}}{z^{10}}\right] = \boxed{\log\left[\frac{\sqrt{xy^3}}{z^{10}}\right]}$$

ex: Condense.

★ c) $-3\log x - 4\log y - \frac{2}{3}\log z$

$-\log x^3 - \log y^4 - \log z^{2/3} \longrightarrow \log\left[\frac{1}{x^3y^4z^{2/3}}\right] = \boxed{\log\left[\frac{1}{x^3y^4\sqrt[3]{z^2}}\right]}$

★ Easier this way

add 1 into the numerator

factor out

$-\left[\log x^3 + \log y^4 + \log z^{2/3}\right]$

$-\log(x^3y^4z^{2/3})$

$-1\log(x^3y^4\sqrt[3]{z^2})$

$\log(x^3y^4\sqrt[3]{z^2})^{-1}$

$\boxed{\log\left(\frac{1}{x^3y^4\sqrt[3]{z^2}}\right)}$

ex: Condense.

d) $\frac{1}{3}\log_4 a + \frac{2}{3}\log_4 b - \frac{4}{3}\log_4 c$

$= \log_4 a^{1/3} + \log_4 b^{2/3} - \log_4 c^{4/3}$

$= \log_4 \left[\frac{a^{1/3} b^{2/3}}{c^{4/3}}\right]$

$= \boxed{\log_4 \sqrt[3]{\frac{ab^2}{c^4}}}$

Change of Base Formula

$$\boxed{\log_b x = \frac{\log_a x}{\log_a b} = \underset{\text{(base 10)}}{\frac{\log x}{\log b}} = \underset{\text{(base e)}}{\frac{\ln x}{\ln b}}}$$

Change of base formula is NOT used to expand or condense

ex: Rewrite using common (Base 10) and natural (Base e) logarithms.
Then evaluate on your calculator. To 3 decimal places.

a) $\log_2 11 = \frac{\log 11}{\log 2} = \frac{\ln 11}{\ln 2} \approx \boxed{3.459}$

Base 10 Base e

ex: Rewrite using common and natural logarithms. Then evaluate on your calculator.

b) $\log_3 25 = \dfrac{\log 25}{\log 3} = \dfrac{\ln 25}{\ln 3} \approx \boxed{2.930}$

ex: Evaluate <u>without</u> the use of a calculator.

★ change of base backwards

a) $\frac{\ln 8}{\ln 2} = \log_2 8 = \boxed{3}$

ex: Evaluate <u>without</u> the use of a calculator.

b) $\dfrac{\log 64}{\log 2} = \log_2 64 = \boxed{6}$

* change of base backwards