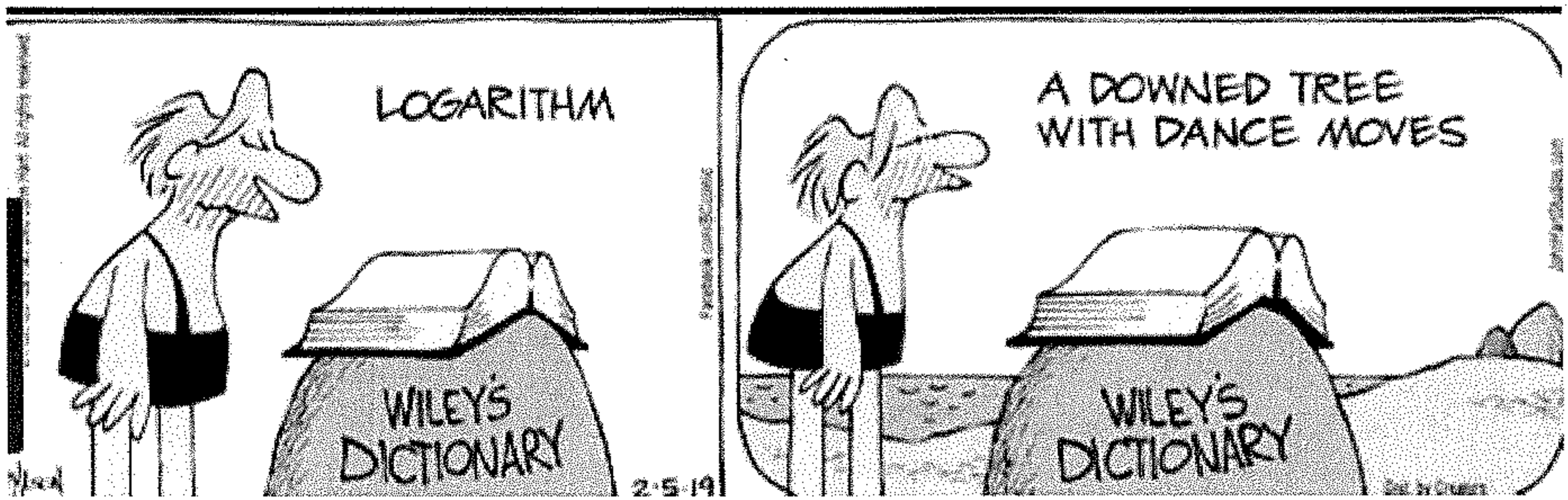
A2: Properties of Logarithms - Day 1

Notes



Logarithms and exponentials are INVERSES!

$$f\left(x\right) = \log_b x$$

$$g\left(x\right) = b^x$$

Evaluate.

$$f(x) = \log_b x \qquad g(x) = b^x$$

a) $(f \circ g)(x) = f(g(x)) = f(b^x) = \log_b(b^x) = \boxed{x}$

b) $(g \circ f)(x) = g(f(x)) = g(\log_b x) = b^{\log_b x} = \boxed{x}$

Property of logs

Inverse properties

$$\log_b b^x = x$$

$$b^{\log_b x} = x$$

$$\ln e^x = x$$

$$e^{\ln x} = x$$

$$e^{\log_e x} = \boxed{x}$$

Evaluate.

a) $7^{\log_7 x} = \boxed{x}$

b) $\log_{62} 62^x = \boxed{x}$

Evaluate.

c) $\log 10^x$

$$\log_{10} 10^x = \boxed{x}$$

d) $e^{\ln 7}$

$$e^{\log_e 7} = \boxed{7}$$

Evaluate.

e) $\log_5 5^4 = \boxed{4}$

f) $\underline{8}^{\log_{\underline{8}} 6} = \boxed{6}$

Logarithm Properties

Let b, m, and n be positive numbers such that $b \neq 1$.

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \cdot \log_b m$

Condenses $\Longleftrightarrow$ Expands

Logarithm properties are used to <u>EXPAND</u> and <u>CONDENSE</u> logarithmic expressions.

Expand. Simplify if possible.

a) $\log_3\left(a^2b^5\right) = \log_3 a^2 + \log_3 b^5$

$= \boxed{2\log_3 a + 5\log_3 b}$

b) $\log_5\left(\dfrac{a^2b^3}{c^4}\right) = \log_5 a^2 + \log_5 b^3 - \log_5 c^4$

$= \boxed{2\log_5 a + 3\log_5 b - 4\log_5 c}$

Expand. Simplify if possible.

c) $\ln\left(e^4x^4\right)$

$= \ln e^4 + \ln x^4$

$\boxed{\ln e = 1}$

$= 4\ln e + 4\ln x$

$= 4(1) + 4\ln x$

$= \boxed{4 + 4\ln x}$

d) $\log_2\left(4xy\right)^2$

$= 2 \cdot \log_2(4xy)$

$= 2[\log_2 4 + \log_2 x + \log_2 y)$

$= 2\log_2 4 + 2\log_2 x + 2\log_2 y$

$\log_2 4 = 2$

$= 2(2) + 2\log_2 x + 2\log_2 y$

$= \boxed{4 + 2\log_2 x + 2\log_2 y}$

Expand. Simplify if possible.

e) $\ln\left(\dfrac{1}{ab^2c^3}\right)$

$$= \ln 1 - \ln a - \ln b^2 - \ln c^3$$

$$= \underbrace{\ln 1}_{0} - \ln a - 2\ln b - 2\ln c$$

$$= \boxed{-\ln a - 2\ln b - 2\ln c}$$

$\boxed{\ln 1 = 0}$ * * *

f) $\log\sqrt{x^5y^6}$

$$= \log(x^5y^6)^{1/2}$$

$$= \frac{1}{2}\log(x^5y^6)$$

$$= \frac{1}{2}\left[\log x^5 + \log y^6\right]$$

$$= \frac{1}{2}\left[5\log x + 6\log y\right]$$

$$= \boxed{\frac{5}{2}\log x + 3\log y}$$

Expand. Simplify if possible.

$$g)\ \ln\left(\frac{x+2}{x}\right) = \boxed{\ln(x+2) - \ln x}$$

no rule
for add/sub.
in the argument.

$$h)\ \log_2 \sqrt[3]{\frac{16a^5}{a+b}} = \log_2\left(\frac{16a^5}{a+b}\right)^{1/3}$$

$$= \frac{1}{3}\log_2\left(\frac{16a^5}{a+b}\right)$$

$$= \frac{1}{3}\left[\log_2 16 + \log_2 a^5 - \log_2(a+b)\right]$$

$\log_2 16 = 4$

$$= \frac{1}{3}\left[4 + 5\log_2 a - \log_2(a+b)\right]$$

$$= \boxed{\frac{4}{3} + \frac{5}{3}\log_2 a - \frac{1}{3}\log_2(a+b)}$$

Expand. Simplify if possible.

i) $\log_3\left(\dfrac{81}{xy^2}\right) = \underbrace{\log_3 81} - \log_3 x - \log_3 y^2$

$= \boxed{4 - \log_3 x - 2\log_3 y}$

$\log_3 81 = 4$

No rule for subtraction in argument

j) $\log_5\left(\dfrac{x^2-16}{125}\right) = \log_5(x^2-16) - \underbrace{\log_5 125}$

$= \log_5(x^2-16) - 3$

DOS factors

$\log_5 125 = 3$

$= \log_5[(x+4)(x-4)] - 3$

now multiplication so... expands

$= \boxed{\log_5(x+4) + \log_5(x-4) - 3}$

Expand. Simplify if possible.

need ( ) here

k) $\log_4\left(\dfrac{16x}{x-4}\right) = \underbrace{\log_4 16} + \log_4 x - \log_4 (x-4)$

$= \boxed{2 + \log_4 x - \log_4 (x-4)}$

Condense. Into one log

a) $2\log_5 a + 3\log_5 b + 4\log_5 c$

$= \log_5 a^2 + \log_5 b^3 + \log_5 c^4$

$= \boxed{\log_5(a^2b^3c^4)}$

b) $3\ln x + 2\ln y - 10\ln z$

$= \ln x^3 + \ln y^2 - \ln z^{10}$

$= \boxed{\ln\left[\frac{x^3y^2}{z^{10}}\right]}$

Subtraction tells us that piece goes into the denominator

Condense.

c) $\log_3(x-3)-\log_3(x+2)$

$$= \boxed{\log_3\left[\frac{x-3}{x+2}\right]}$$

d) $\log x+\log(x+1)$

$$\boxed{\log[x(x+1)]} \text{ or } \boxed{\log(x^2+x)}$$

Condense.

e) $\log x - 2\log(x+5) - \frac{1}{2}\log(x-1)$

$= \log x - \log(x+5)^2 - \log(x-1)^{1/2}$

$= \boxed{\log\left[\frac{x}{(x+5)^2(x-1)^{1/2}}\right]}$ or $\boxed{\log\left[\frac{x}{(x+5)^2\sqrt{x-1}}\right]}$

f) $\frac{1}{2}\log x - \frac{1}{2}\log y - \frac{3}{2}\log z$

$= \log x^{1/2} - \log y^{1/2} - \log z^{3/2}$

$= \log\left[\frac{x^{1/2}}{y^{1/2}z^{3/2}}\right] = \log\left[\frac{\sqrt{x}}{\sqrt{y}\cdot\sqrt{z^3}}\right] = \log\sqrt{\frac{x}{yz^3}}$

all square roots... so can combine them.