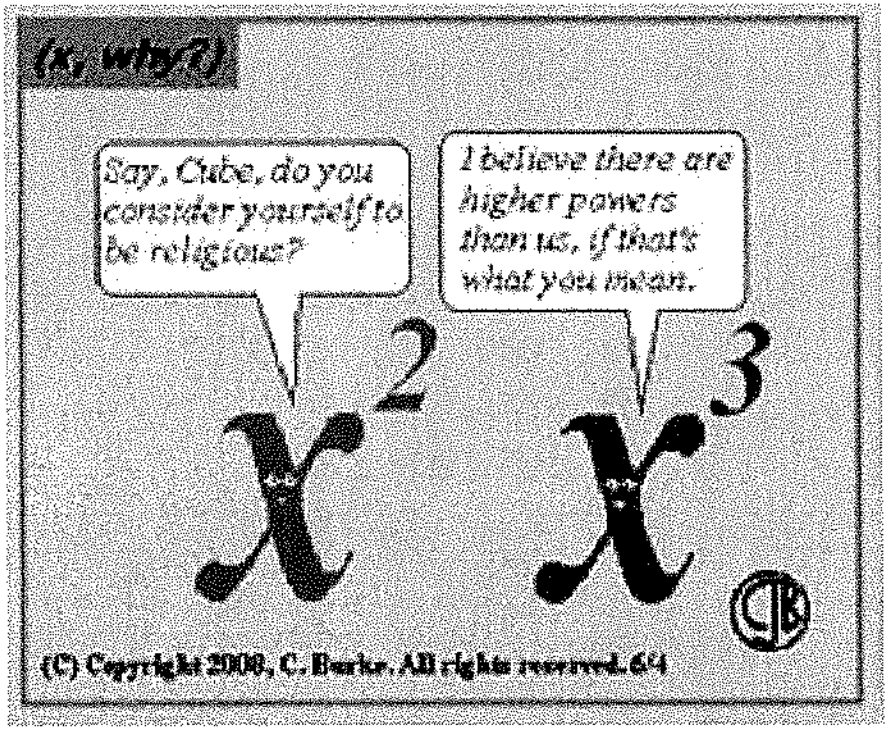
Notes

Properties of Exponents
Introduction to Polynomial Functions
Polynomial Operations - Addition and Subtraction



REVIEW: Exponent Properties

Property Name	Definition
Product of Powers	$a^m \cdot a^n = a^{m+n}$ (add)
Power of a Power	$(a^m)^n = a^{mn}$ (mult.)
Power of a Product	$(ab)^m = a^m b^m$ (mult)
Negative Exponent	$a^{-m} = \frac{1}{a^m}$
Zero Exponent	$a^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$ (subtract)
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

ex: Simplify.

a) $x^2 \cdot x^7 = x^{2+7} = \boxed{x^9}$

b) $(5x^7)^2 = (5)^2 (x^{7})^{2}$ (mult)

$= \boxed{25x^{14}}$

Answer:

- No negative exponents
- No "0" exponents

ex: Simplify.

c) $\frac{x^2}{x^7} = x^{2-7} = \frac{x^{-5}}{1} = \boxed{\frac{1}{x^5}}$

$\frac{x^2 \cdot 1}{x^7 \cdot x^{-2}} = \boxed{\frac{1}{x^5}}$

$\frac{x \cdot x \cdot 1}{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}$

d) $\frac{3x^{-2}}{x^7} = \frac{3}{x^2 \cdot x^7} = \boxed{\frac{3}{x^9}}$

not $(3x)^{-2} = \frac{1}{(3x)^2} = \boxed{\frac{1}{9x^2}}$

not Add/Sub.

$\frac{3 + x^{-2}}{x^7} \neq$

ex: Simplify.

e) $\left(2x^5\right)\left(5x^{-4}\right)^3\left(3x^0\right)$

$(5)^3(x^{-4})^3$

$2x^5 \cdot 125x^{-12} \cdot 3 = \dfrac{2 \cdot 125 \cdot 3\, x^5}{x^{12}}$ (250)

$750x^{-7} = \dfrac{750x^5}{x^{12-7}}$

$= \boxed{\dfrac{750}{x^7}}$

$25 \times 5 = 125$

ex: Simplify.

f) $\dfrac{3x^2y^2}{2x^{-1}\cdot(4x^2y)^3}$

$(4)^3(x^2)^3(y)^3$

$16 \times 4 = 64$

(1) Can you simplify the numerator?

(2) Can you simplify the denominator?

(3) negative exponents?

(4) "reduce" btwn N & D.

$\dfrac{3x^2y^2}{2x^{-1}\cdot 64x^6y^3}$

$\dfrac{3x^2y^2}{128x^{5}y^{3}}$

42
$3\overline{)128}$
$-12\downarrow$
08
-6
$2 \leftarrow R$

$\dfrac{3}{128x^3y}$

ex: Simplify.

g) $\dfrac{\left(2x^3z^0\right)^3}{\left(x^3y^{-4}z^2\right)\left(x^4z^{-3}\right)} = \dfrac{(2)^3(x^3)^3}{x^7y^{-4}z^{-1}} = \dfrac{8x^9}{x^7y^{-4}z^{-1}}$

$x^3 \cdot x^4 = x^7$

$z^2 \cdot z^{-3} = z^{-1}$

$= \dfrac{8x^9y^4z^1}{x^7}$

$= \boxed{8x^2y^4z}$

Monomial - a number, a variable or a product of numbers and variables

↑ multiplication

one term

9

$9x$

x^2

$5xy^2$

Polynomial - an expression involving one or more monomials

added or subtracted together

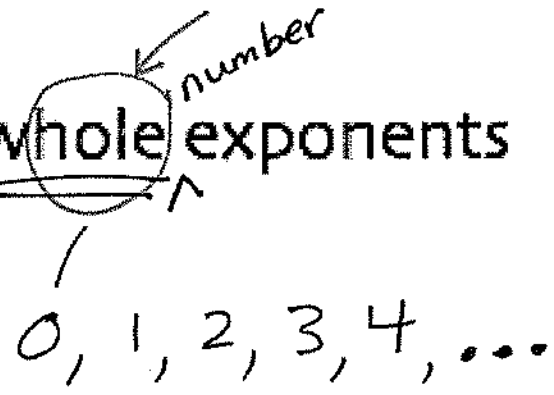
9 (1 term)

$9 + 5x$ (2 terms)

$3x^2 - 2x + 1$ (3 terms)

Characteristics of Polynomials

1. variables have whole exponents



2. real coefficients

no i

3. no division by variables

- no variables in the denominator

ex: Determine whether the expression represents a polynomial.

a) $2x^2 - 4x^1 + \frac{1}{7}$ yes

(3 terms)

constant: $\frac{1}{7}$

$\frac{1}{7}x^0$

0

$0x^0$

b) 0 yes

Constant ↗

(1 term)

ex: Determine whether the expression represents a polynomial.

(1 term)

c) $\frac{5}{x^2}$ ← NO b/c can't have a variable in the denominator

$= 5x^{-2}$ ↖ not a whole #

d) $\frac{x^2}{5}$ YES

(1 term)

$\frac{1}{5}x^2$

Classifying Polynomials

1. Degree - largest exponent
(on the variable)

"First name"

Degree	Type
0	constant
1	linear
2	quadratic
3	cubic
4	quartic
5	quintic
≥6	nth degree polynomial

think "quarter"

n = degree

Classifying Polynomials

"Last name"

2. Number of terms

Number of Terms	Type
1	monomial
2	binomial
3	trinomial
≥4	polynomial

ex: State the degree and number of terms. Then classify.

a) $4x - 27x^2 + 3$

Reorder: $-27x^{2} + 4x + 3$

Degree: 2

# of terms: 3

Quadratic trinomial

b) $3x + 7$

$3x^{1} + 7$

Degree: 1

# of terms: 2

Linear binomial

ex: State the degree and number of terms. Then classify.

c) $5x^{6}+2x^{3}+4x-5$

Degree: 6

# of terms: 4

6th degree polynomial

d) $5x^{4}$

Degree: 4

# of terms: 1

Quartic monomial

e) -8

$-8 \cdot x^{0}$

Degree: 0

# of Terms: 1

Constant monomial

Standard Form of a Polynomial - a polynomial is in standard form when the terms' exponents are in descending order.

ex: Write the polynomial in standard form.

$$1+2x-3x^4$$

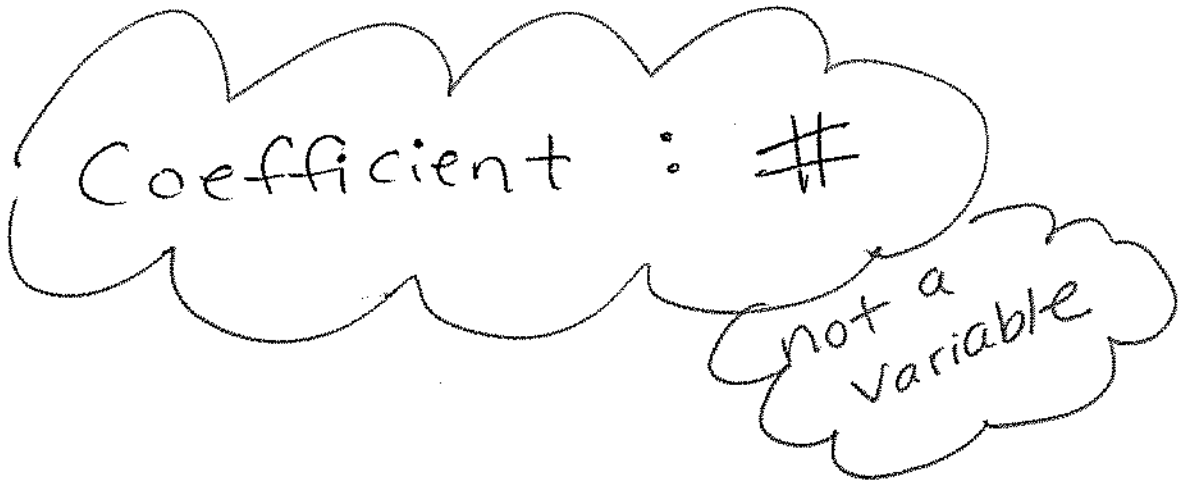
standard form → $-3x^4+2x+1$ descending order

Degree: 4

# of terms: 3

Quartic trinomial

Leading Coefficient - the coefficient of the term that defines the degree



ex: Identify the leading coefficient.

$$\frac{1}{5}x - 3x^4 + 10$$

reorder: $-3x^4 + \frac{1}{5}x + 10$ Standard form

↑

Lead. coeff: -3

ex: Consider the four polynomial functions.

$a(x) = -5$

$b(x) = 5x^4 + 2$

$c(x) = 5x^2 + 4x - 3$

$d(x) = 2x - 1$

Perform the indicated operation. Write the answer in standard form.

a) $a(x) + b(x)$ add

$(-5) + (5x^4 + 2)$ add

now drop the parenthesis

$-5 + 5x^4 + 2$

$5x^4 - 3$ standard form

ex: Consider the four polynomial functions.

$a(x) = -5$

$b(x) = 5x^4 + 2$

$c(x) = 5x^2 + 4x - 3$

$d(x) = 2x - 1$

Perform the indicated operation. Write the answer in standard form.

b) $b(x) - c(x)$ Subtract

$(5x^4 + 2) - (5x^2 + 4x - 3)$ Subtract

now drop the parenthesis

$5x^4 + 2 - 5x^2 - 4x + 3$

$5x^4 - 5x^2 - 4x + 5$ standard form

ex: Consider the four polynomial functions.

$$a(x) = -5 \qquad b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3 \qquad d(x) = 2x - 1$$

Perform the indicated operation. Write the answer in standard form.

c) $8c(x)$

$8 \cdot c(x)$

$8(5x^2 + 4x - 3)$

$40x^2 + 32x - 24$

ex: Consider the four polynomial functions.

$$a(x) = -5$$

$$b(x) = 5x^4 + 2$$

$$c(x) = 5x^2 + 4x - 3$$

$$d(x) = 2x - 1$$

Perform the indicated operation. Write the answer in standard form.

d) $d(x) - 5b(x)$

$d(x) - 5 \cdot b(x)$

$(2x-1) - 5(5x^4+2)$

$2x - 1 - 25x^4 - 10$

$-25x^4 + 2x - 11$