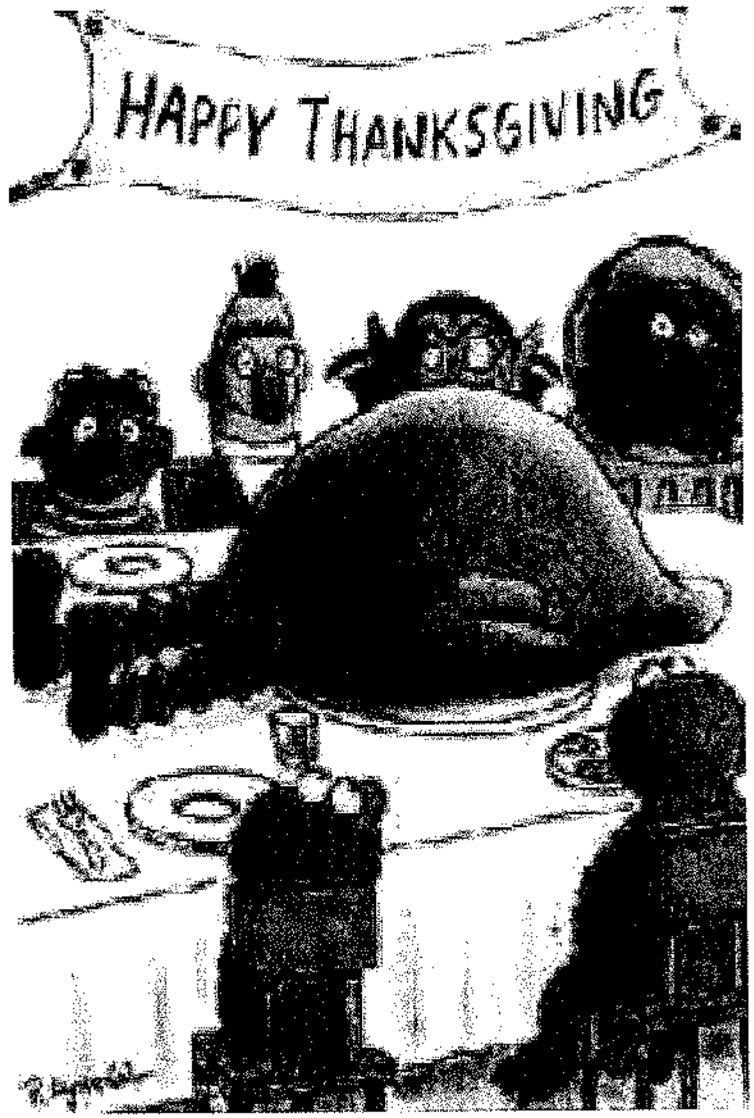
Polynomial Division

Notes



Polynomial Division Techniques

1. Dividing By A Monomial one term

2. Dividing By A Polynomial

   - Long Division
   - Synthetic Division

1. Dividing By A Monomial

ex: Divide.

$$\frac{7x^4 - 5x^2 + 14x}{21x^3}$$ ← one term in the denominator

Split it up!

$$\frac{\overset{1}{\cancel{7}}\,x^{\cancel{4}\,1}}{\underset{3}{\cancel{21}}\,\cancel{x^3}} - \frac{5\cancel{x^2}}{21x^{\cancel{3}\,1}} + \frac{\overset{2}{\cancel{14}}\,\cancel{x}}{\underset{3}{\cancel{21}}\,x^{\cancel{3}\,2}}$$

Reduce each term.

$$\boxed{\frac{x}{3} - \frac{5}{21x} + \frac{2}{3x^2}}$$

2. Dividing By A Polynomial - Long Division

REVIEW: Divide.

$$\begin{array}{r} 1309 \\ 4\overline{)5239} \\ \underline{-4} \\ 12 \\ \underline{-12} \\ 039 \\ \underline{-36} \\ 3 \end{array}$$

Divisor

3 ← Remainder

$$\boxed{1309\frac{3}{4}}$$

R

Divisor

ex: Divide using long division.

a) $\dfrac{x^2 + 2x + 7}{x - 8}$

No missing terms

mult.

$$
\begin{array}{r}
x + 10 \\
x-8 \overline{) \, x^2 + 2x + 7} \\
-(x^2 - 8x) \\
\hline
10x + 7 \\
-(10x - 80) \\
\hline
+87
\end{array}
$$

← R

subtract (all signs → change)

$x(x-8) = x^2 - 8x$

$10(x-8) = 10x - 80$

$$\boxed{x + 10 + \frac{87}{x-8}}$$

Remainder

Divisor

ex: Divide using long division.

b) $\dfrac{x^2+3x-40}{x+5}$

no missing terms

$$
\begin{array}{r|l}
 & x - 2 \\
\hline
x+5 & \cancel{x^2} + 3x - 40 \\
 & -\cancel{x^2} - 5x \quad \downarrow \\
\hline
 & \quad -\cancel{2x} - 40 \\
 & \quad +\cancel{2x} + 10 \\
\hline
 & \qquad\quad -30 \leftarrow R
\end{array}
$$

$x(x+5) = x^2+5x$

$-2(x+5) = -2x-10$

$$\boxed{x - 2 - \frac{30}{x+5}}$$

Can <u>not</u> reduce here because of the <u>x+5</u>.

ex: Divide using long division.

c) $\dfrac{6x^3-11x^2+14x-11}{2x-1}$

no missing terms

$$\begin{array}{r} 3x^2-4x+5 \\ 2x-1\overline{)\,6x^3-11x^2+14x-11} \\ \underline{-6x^3+3x^2} \\ -8x^2+14x \\ \underline{+8x^2-4x} \\ 10x-11 \\ \underline{-10x+5} \\ -6 \leftarrow R \end{array}$$

$3x^2(2x-1) = 6x^3-3x^2$

$-4x(2x-1) = -8x^2+4x$

$5(2x-1) = 10x-5$

$$\boxed{3x^2-4x+5-\frac{6}{2x-1}}$$

Does not reduce.

ex: Divide using long division.

d) $\dfrac{5x^3 - 2x^2 + 1}{x+3}$

add a place holder (missing term)

$$\begin{array}{r} 5x^2 - 17x + 51 \\ x+3 \overline{\smash{\big)}\, 5x^3 - 2x^2 + 0x + 1} \\ \underline{-5x^3 - 15x^2 } \\ -17x^2 + 0x \\ \underline{+17x^2 + 51x } \\ 51x + 1 \\ \underline{-51x - 153} \\ -152 \leftarrow R \end{array}$$

$5x^2(x+3) = 5x^3 + 15x^2$

$-17x(x+3) = -17x^2 - 51x$

$51(x+3) = 51x + 153$

$$\boxed{5x^2 - 17x + 51 - \frac{152}{x+3}}$$

2. Dividing By A Polynomial - Synthetic Division

Can only divide
by a linear binomial
w/ leading coefficient = 1

ex. $x+5$

$x+2$

$x-3$

ex: Divide using synthetic division.

a) $\dfrac{x^2+2x+7}{x-8}$

linear binomial w/ leading coeff = 1

* Add place holders for missing terms (MT's) (none here).

Degree 2

$x^2 + 2x + 7$

mult by box

$$
\begin{array}{c|ccc}
8 & 1 & +2 & +7 \\
 & \downarrow & 8 & 80 \\
\hline
 & 1 & +10 & +87 \leftarrow R
\end{array}
$$

add columns

$x - 8 = 0$

$x = 8$

Put this in the box.

now Degree 1

$x^1 + 10$

$$\boxed{x + 10 + \frac{87}{x-8}}$$

R

divisor

ex: Divide using synthetic division.

b) $\dfrac{x^2+3x-40}{x+5}$

• no missing terms

Degree 2

$1x^2 + 3x - 40$

mult

$x+5=0$

$x=-5$

$$
\begin{array}{r|rrr}
-5 & 1 & +3 & -40 \\
 & \downarrow & -5 & 10 \\
\hline
 & 1 & -2 & -30 \\
\end{array}
\leftarrow R
$$

add columns

now Degree: 1

$1x^1 - 2$

$$\boxed{x - 2 - \frac{30}{x+5}}$$

ex: Divide using synthetic division.

c) $\dfrac{x^2-4}{x-1}$

add a place holder (MT)

$1x^2 + 0x - 4$ add

$$\begin{array}{r|rrr} 1 & 1 & 0 & -4 \\ & \downarrow & 1 & 1 \\ \hline & 1 & 1 & -3 \end{array} \leftarrow R$$

mult.

$x-1=0$

$x=1$

now Degree: 1

$1x^1 + 1$

$$\boxed{x+1-\frac{3}{x-1}}$$

ex: Divide using any technique.

a) $\dfrac{x^3+2x^2+2x+9}{x^2+5}$

add place holder

• not a linear binomial w/coeff = 1

so must use long division

$$\begin{array}{r} x+2 \\ x^2+0x+5 \overline{\smash{)}\, x^3+2x^2+2x+9} \\ \underline{\cancel{x^3}\ \cancel{+}0x^2\ \cancel{+}5x\ \downarrow} \\ \cancel{2x^2}-3x+9 \\ \underline{\cancel{2x^2}\ \cancel{+}0x\ \cancel{+}10} \\ -3x-1 \leftarrow R \end{array}$$

factor

$$\boxed{x+2+\frac{-3x-1}{x^2+5}}$$

$$* \quad \boxed{x+2-\frac{3x+1}{x^2+5}}$$

ex: Divide using any technique.

b) $\dfrac{x^3 + 7x^2 - x}{x + 3}$

add missing term (constant)

use sythetic w/ linear binomial w/ coeff = 1

degree 3

MT

$1x^3 + 7x^2 - 1x + \underline{0}$

$x+3=0$
$x=-3$

$$\begin{array}{r|rrrr} -3 & 1 & +7 & -1 & 0 \\ & \downarrow & -3 & -12 & +39 \\ \hline & 1 & +4 & -13 & \multicolumn{1}{|r}{+39} \end{array} \leftarrow R$$

now degree: 2

$1x^2 + 4x - 13$

$$\boxed{x^2 + 4x - 13 + \frac{39}{x+3}}$$

can not reduce here.

Evaluating Polynomials

There are two ways to evaluate polynomial functions:

1. direct substitution • old way (plug in)

2. synthetic substitution • new way

Direct Substitution (i.e. "PLUG IN")

ex: Find the indicated polynomial value using direct substitution.

a) $f(x) = x^2 - 5x + 2, \quad f(\underset{\substack{\uparrow \\ x=13}}{13}) = ?$

$$f(13) = (13)^2 - 5(13) + 2$$

$$= \underbrace{169 - 65}_{} + 2$$

$$= 104 + 2$$

$$\boxed{f(13) = 106}$$

ex: Find the indicated polynomial value using direct substitution.

b) $g(x) = x^3 + 4x^2 - 1, \qquad g(6) = ?$

$x = 6$

$$g(6) = (6)^3 + 4(6)^2 - 1$$

$$= 216 + 4(36) - 1$$

$$= \underbrace{216 + 144} - 1$$

$$= 360 - 1$$

$$\boxed{g(6) = 359}$$

Synthetic Substitution - substitution using a chart of coefficients

• uses synthetic division

*Before using synthetic substitution,
- the polynomial must be in standard form
- consider if all terms are present missing terms?

ex: Find the indicated value using synthetic substitution.

new

a) $f(x) = x^2 - 5x + 2, \quad f(13) = ?$

$x = 13$

• no missing terms ✓
• in descending order ✓

$1x^2 - 5x + 2$

$$\begin{array}{r|rrr} 13 & 1 & -5 & +2 \\ & \downarrow & +13 & +104 \\ \hline & 1 & +8 & \boxed{106} \leftarrow R \end{array}$$

$$\begin{array}{r} {}^{2}13 \\ \times 8 \\ \hline 104 \end{array}$$

R

$\boxed{f(13) = 106}$

ex: Find the indicated value using synthetic substitution.

b) $g(x) = x^3 + 4x^2 - 1, \quad g(6) = ?$

add MT

$x = 6$

$$1x^3 + 4x^2 + \overset{MT}{0x} - 1$$

$$\begin{array}{r|rrrr} 6 & 1 & +4 & 0 & -1 \\ & \downarrow & +6 & +60 & +360 \\ \hline & 1 & +10 & +60 & +359 \end{array} \leftarrow R$$

$$\boxed{g(6) = 359}$$

R