n^{th} Roots - Multiplying, Dividing, Rationalizing

Notes

$$\frac{\sqrt{\cancel{2}}}{\cancel{2}} = \sqrt{}$$

Operations with n^{th} Roots: Multiplication

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers,

then $\sqrt[n]{a} \cdot \sqrt[n]{b} \Leftrightarrow$ $\underline{\sqrt[n]{a \cdot b}}$.

*The indexes must be the same when multiplying.

ex: Can you simplify the product of the radical expressions? Explain.

a) $\sqrt[3]{6} \cdot \sqrt{2}$ no, different indexes

b) $\sqrt[3]{-4} \cdot \sqrt[3]{2}$ yes, same indexes

ex: Multiply, if possible. Then simplify.

a) $\sqrt[3]{5} \cdot \sqrt[3]{-25}$

$$= \sqrt[3]{5 \cdot (-25)}$$

$$= \sqrt[3]{-1 \cdot 5 \cdot 25} \rightarrow 5 \qquad 5 \overline{) 25} \quad 5$$

$$= \sqrt[3]{-1 \cdot 5 \cdot 5 \cdot 5}$$

$$= -1 \cdot 5$$

$$= \boxed{-5}$$

ex: Multiply, if possible. Then simplify.

b) $\sqrt[4]{18} \cdot \sqrt[4]{6}$

$= \sqrt[4]{18 \cdot 6}$

$= \sqrt[4]{2 \cdot 3 \cdot 3 \cdot 2 \cdot 3}$

$= \boxed{\sqrt[4]{108}}$

$18 = 2 \cdot 9,\ 9 = 3 \cdot 3$ $\quad$ $6 = 2 \cdot 3$

$$\begin{array}{r} 18 \\ \times\ 6 \\ \hline 108 \end{array}$$

What if it had an index of 3?

$\sqrt[3]{18} \cdot \sqrt[3]{6} = \sqrt[3]{18 \cdot 6}$

$= 3\sqrt[3]{2 \cdot 2}$

$= \boxed{3\sqrt[3]{4}}$

$18 = 2 \cdot 9,\ 9 = 3 \cdot 3$ $\quad$ $6 = 2 \cdot 3$

ex: Multiply, if possible. Then simplify.

c) $\sqrt[4]{8} \cdot \sqrt[3]{32}$

* not possible to multiply b/c different indexes, but we can simplify the radicand(s)

$$\sqrt[4]{8} \cdot 2\sqrt[3]{2 \cdot 2}$$

$$\boxed{\sqrt[4]{8} \cdot 2\sqrt[3]{4}}$$

or

$$\boxed{2\sqrt[4]{8}\sqrt[3]{4}}$$

$$\cancel{\begin{array}{r|l} 2 & 8 \\ 2 & 4 \\ & 2 \end{array}}$$

$$\begin{array}{r|l} 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ \underline{\underline{2}} & 4 \\ & \underline{\underline{2}} \end{array}$$

ex: Multiply, if possible. Then simplify.

d) $\sqrt[3]{6} \cdot \sqrt[3]{16}$

$$= \sqrt[3]{6 \cdot 16}$$

$$= 2\sqrt[3]{2 \cdot 3 \cdot 2}$$

$$= \boxed{2\sqrt[3]{12}}$$

$2 \,|\, 6$
$\quad 3$

$2 \,|\, 16$
$2 \,|\, 8$
$2 \,|\, 4$
$\quad 2$

Operations with n^{th} Roots: Division

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers,

then $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$.

*The indexes must be the same when dividing.

ex: Divide, if possible. Then simplify.

a) $\dfrac{\sqrt{18}}{\sqrt{2}} = \sqrt{\dfrac{18}{2}} = \sqrt{9} = \boxed{3}$

simplify 1st

$\dfrac{\sqrt{18}}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{18 \cdot 2}}{\sqrt{4}} = \dfrac{\cancel{2} \cdot 3}{\cancel{2}} = \boxed{3}$

NOT the best method here.

$2 \,|\, 18$
$3 \,|\, 9$
3

2

ex: Divide, if possible. Then simplify.

b) $\dfrac{\sqrt[3]{162}}{\sqrt[3]{3}} = \sqrt[3]{\dfrac{162}{3}} = \sqrt[3]{54}$

$= \boxed{3\sqrt[3]{2}}$

$$\begin{array}{r|l} 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

$$\begin{array}{r} 54 \\ 3\overline{)162} \\ -15\downarrow \\ \hline 12 \\ -12 \\ \hline 0 \end{array}$$

ex: Perform the indicated operation. Give your answer in simplest form.

a) $\sqrt{2}\cdot\left(\sqrt{50}+7\right)$

multiply (distribute)

$$= \sqrt{2\cdot 50} + 7\sqrt{2}$$

$$= \sqrt{2\cdot 2\cdot 5\cdot 5}$$

$$= \underbrace{2\cdot 5} + 7\sqrt{2}$$

$$= \boxed{10 + 7\sqrt{2}} \neq 17\sqrt{2}$$

No! No! No!

Final answer.

NOT like terms.

$$\begin{array}{r|l} 2 & 50 \\ 5 & 25 \\ & 5 \end{array}$$

2

ex: Perform the indicated operation. Give your answer in simplest form.

b) $\left(1-\sqrt[5]{2}\right)\left(2+\sqrt[5]{2}\right) = 2 + \underbrace{1\sqrt[5]{2} - 2\sqrt[5]{2}}_{\text{like radicals}} - 1\sqrt[5]{2\cdot 2}$

$- 1\sqrt[5]{4}$

FOIL

$= 2 - 1\sqrt[5]{2} - 1\sqrt[5]{4}$

$= \boxed{2 - \sqrt[5]{2} - \sqrt[5]{4}}$

ex: Perform the indicated operation. Give your answer in simplest form.

c) $\left(3+\sqrt[3]{4}\right)^2 = \left(3+\sqrt[3]{4}\right)\left(3+\sqrt[3]{4}\right)$

Foil

$2\,\frac{4}{2} \quad 2\,\frac{4}{2}$

$= 9 + \underbrace{3\sqrt[3]{4} + 3\sqrt[3]{4}}_{\text{like radicals}} + 1\sqrt[3]{4\cdot 4}$

$+ 2\sqrt[3]{2}$

$= \boxed{9 + 6\sqrt[3]{4} + 2\sqrt[3]{2}}$

Rationalizing Denominators

A denominator is rationalized when no radicals exist in the denominator.

ex: Rationalize the denominator.

a) $$\frac{2}{\sqrt{3}} \cdot \underbrace{\frac{\sqrt{3}}{\sqrt{3}}}_{1} = \frac{2\sqrt{3}}{\sqrt{9}} = \boxed{\frac{2\sqrt{3}}{3}}$$

index = 2

What if:

$$\frac{\overset{2}{\cancel{6}}\sqrt{3}}{\underset{1}{\cancel{3}}} = \boxed{2\sqrt{3}}$$

Simplify here.

ex: Rationalize the denominator.

b) $\frac{2}{\sqrt[3]{3}} \cdot \underbrace{\frac{\sqrt[3]{3\cdot 3}}{\sqrt[3]{3\cdot 3}}}_{1} = \frac{2\sqrt[3]{3\cdot 3}}{\sqrt[3]{3\cdot 3\cdot 3}} = \boxed{\frac{2\sqrt[3]{9}}{3}}$

ex: Rationalize the denominator.

c) $\frac{1}{\sqrt[4]{2}} \cdot \frac{\sqrt[4]{2\cdot2\cdot2}}{\sqrt[4]{2\cdot2\cdot2}} = \frac{\sqrt[4]{2\cdot2\cdot2}}{\sqrt[4]{2\cdot2\cdot2\cdot2}} = \boxed{\frac{\sqrt[4]{8}}{2}}$

or work w/exponents

1

$\frac{1}{\sqrt[4]{2^1}} \cdot \frac{\sqrt[4]{2^3}}{\sqrt[4]{2^3}} = \frac{\sqrt[4]{2^3}}{\cancel{\sqrt[4]{2^4}}} = \boxed{\frac{\sqrt[4]{8}}{2}}$

$2^1 \cdot 2^3 = 2^4$

ex: Rationalize the denominator.

f) $$\frac{15}{\sqrt[5]{25}} = \frac{15}{\sqrt[5]{5^2}} \cdot \frac{\sqrt[5]{5^3}}{\sqrt[5]{5^3}} = \frac{15\sqrt[5]{5^3}}{\sqrt[5]{5^5}} = \frac{\overset{3}{\cancel{15}}\sqrt[5]{5^3}}{\underset{1}{\cancel{5}}} = \boxed{3\sqrt[5]{125}}$$

Simplify here

ex: Rationalize the denominator.

g) $$\frac{-8}{\sqrt[10]{16}} = \frac{-8}{\sqrt[10]{2^4}} \cdot \frac{\sqrt[10]{2^6}}{\sqrt[10]{2^6}} = \frac{-8\sqrt[10]{2^6}}{\cancel{\sqrt[10]{2^{10}}}} = \frac{\overset{-4}{\cancel{-8}}\sqrt[10]{64}}{\underset{1}{\cancel{2}}} = \boxed{-4\sqrt[10]{64}}$$ *

or

$$\frac{-8}{\sqrt[10]{2^4}} \rightarrow \frac{-8}{\sqrt[5]{2^2}} \cdot \frac{\sqrt[5]{2^3}}{\sqrt[5]{2^3}} = \frac{-8\sqrt[5]{2^3}}{\cancel{\sqrt[5]{2^5}}} = \frac{\overset{-4}{\cancel{-8}}\sqrt[5]{8}}{\underset{1}{\cancel{2}}}$$

$$= \boxed{-4\sqrt[5]{8}}$$

ex: Perform the indicated operation. Rationalize the denominator when necessary.

a) $$\frac{5}{(5-2\sqrt{3})}\cdot\frac{(5+2\sqrt{3})}{(5+2\sqrt{3})} = \frac{5(5+2\sqrt{3})}{25+\cancel{10\sqrt{3}}-\cancel{10\sqrt{3}}-4\sqrt{9}}$$

FOIL

$$-4(3)$$
$$-12$$

2 parts

→ need to use a conjugate (middle sign must be opposite)

$$= \frac{5(5+2\sqrt{3})}{13}$$

$$= \boxed{\frac{25+10\sqrt{3}}{13}}$$