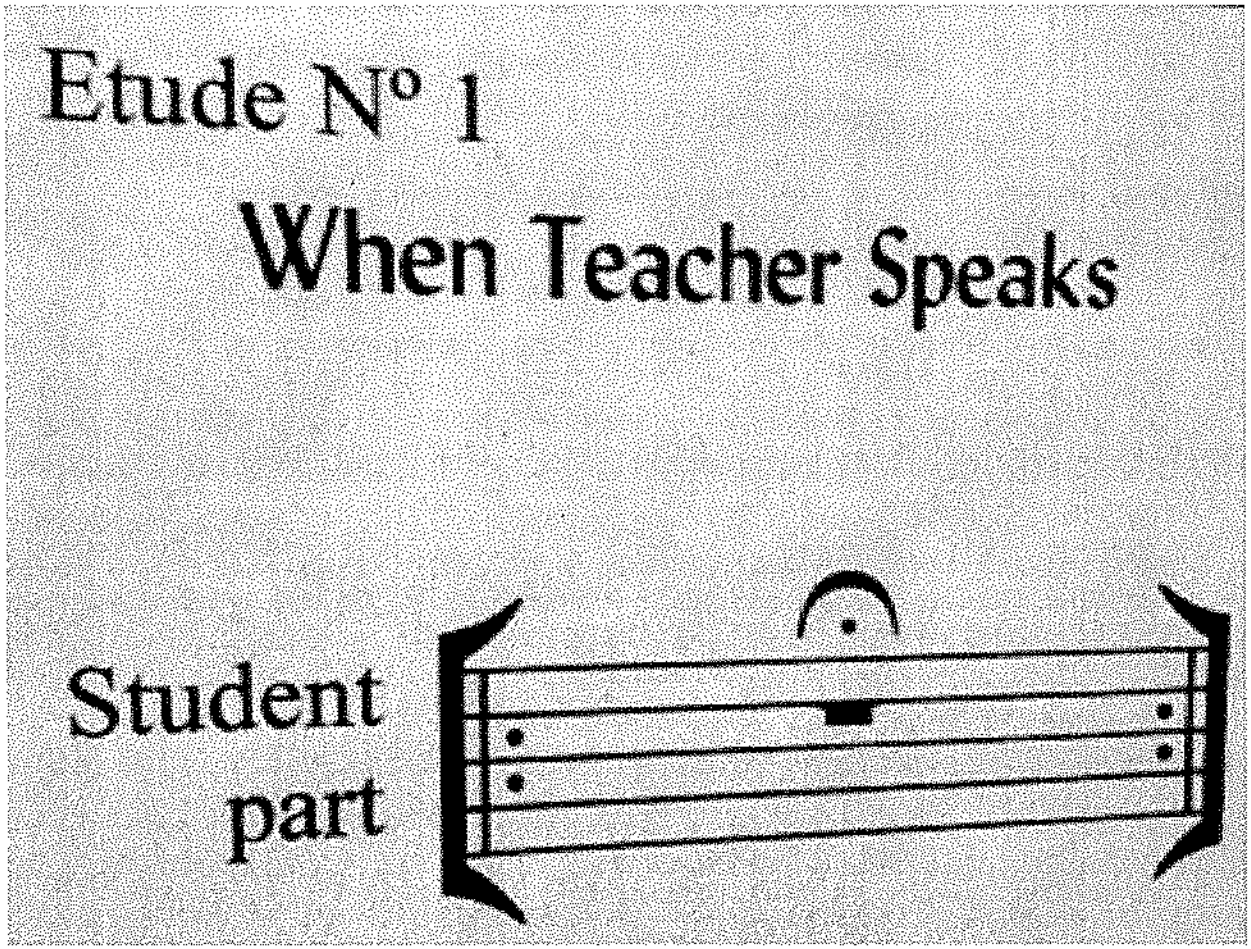
Notes

Multiplying Polynomials

Factoring Bootcamp 2



REVIEW

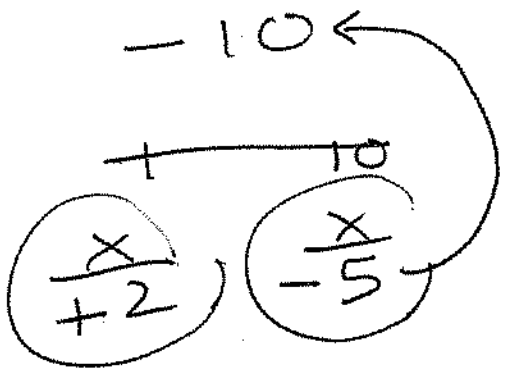
ex: Factor completely.

sum
x^2+9
prime

a) $x^2-9 = (x+3)(x-3)$

no gcf DOS

b) $x^2-3x-10$



no gcf
$(x+2)(x-5)$

REVIEW

ex: Factor completely.

c) $9x^2 + 12x + 4$

perfect trinomial square (PST)

$+36$

$\frac{9x}{+6}$ $\frac{9x}{+6}$ $\div 3$

$\div 3$ reduce each

$\frac{3x}{+2}$ $\frac{3x}{+2}$

no gcf $(3x+2)(3x+2)$

or

$(3x+2)^2$

d) $2x^2 + 32$

gcf $2(x^2 + 16)$

Prime "SOS"

REVIEW

ex: Factor completely.

16 × 3 = 48

e) $x^3 - 3x^2 - 16x + 48$ (no GCF for all 4)

4 terms → Grouping

$x^2(x-3) - 16(x-3)$

$(x-3)(x^2-16)$ → DOS

$\boxed{(x-3)(x+4)(x-4)}$

f) $x^3 + 3x^2 - 2x - 6$ (no GCF for all 4)

$x^2(x+3) - 2(x+3)$

$\boxed{(x+3)(x^2-2)}$

Perfect Cubes

$1^3 = 1$

$2^3 = 8$

$2 \cdot 2 \cdot 2$

$4 \cdot 2$

$3^3 = 27$

$4^3 = 64$

$5^3 = 125$

$6^3 = 216$

$10^3 = 1000$

Factoring
Sum/Difference of Cubes

Rules! Must know!

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

sum

S O AP

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

difference

Binomial Trinomial

Remembering the SIGNS in the Cubes Formula

SOAP

S - same sign

O - opposite sign

AP - always positive

ex: Factor completely.

no gcf

a) $x^3 + 27 = (a + b)(a^2 - ab + b^2)$

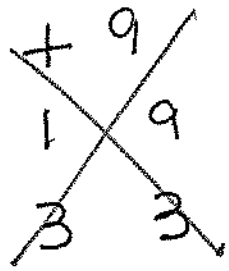
$a = x \quad a^2 = x^2$

$b = 3 \quad ab = 3x$

$b^2 = 9$

S O AP

$= \boxed{(x+3)(x^2 - 3x + 9)}$



no gcf

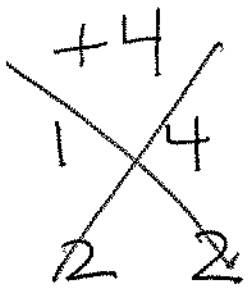
b) $x^3 - 8 = \boxed{(x - 2)(x^2 + 2x + 4)}$

S O AP

$a = x \quad a^2 = x^2$

$b = 2 \quad ab = 2x$

$b^2 = 4$



The trinomial from a cube will always be prime; as long as you factored the GCF 1st.

ex: Factor completely.

c) $64 - x^3 = (4 - x)(16 + 4x + x^2)$

S O AP

no gcf

$a = 4 \quad a^2 = 16$

$b = x \quad ab = 4x$

$b^2 = x^2$

* d) $8x^3 + 125 = (2x + 5)(4x^2 - 10x + 25)$

S O AP

no gcf

$a = 2x \quad a^2 = 2x \cdot 2x = 4x^2$

$b = 5 \quad ab = 2x \cdot 5 = 10x$

$b^2 = 25$

↑ Prime trinomial

~~not~~ PST!

$(2x-5)(2x-5)$

$4x^2 - 10x - 10x + 25$

$-20x$ NO!

ex: Factor completely.

e) $125x^3 - 27 = (5x - 3)(25x^2 + 15x + 9)$

S O AP

no gcf

$a = 5x \quad a^2 = 5x \cdot 5x = 25x^2$

$b = 3 \quad ab = 5x \cdot 3 = 15x$

$b^2 = 9$

**

f) $24x^3 + 3$ GCF 1st!!!

GCF 3

$3(8x^3 + 1) = 3(2x + 1)(4x^2 - 2x + 1)$

S O AP

$a = 2x \quad a^2 = 2x \cdot 2x = 4x^2$

$b = 1 \quad ab = 2x$

$b^2 = 1$

Quadratic in Form

double

$$ax^{2n} + bx^{n} + c$$

ex: Factor completely.

true Quadratic:

a) $x^2+7x+10$

+10 ↖

$\frac{x}{+2}$ $\frac{x}{+5}$

no gcf

$(x+2)(x+5)$

b) x^4+7x^2+10 (double)

Quadratic in Form Trinomial

+10 ↖

$\frac{x^2}{+2}$ $\frac{x^2}{+5}$

$(x^2+2)(x^2+5)$

Foil to check ...

$x^4+5x^2+2x^2+10$

x^4+7x^2+10 ✓

ex: Factor completely.

no gcf

c) $x^4 - 3x^2 - 4$

double

-4

$x^2 \quad x^2$

$+1 \quad -4$

$(x^2+1)(x^2-4)$

DOS

$\boxed{(x^2+1)(x+2)(x-2)}$

* d) $2x^4 + 7x^2 + 6$

no gcf

$+12$

$2x^2 \quad 2x^2$

$+3 \quad +4$

$\div 2$ Reduce

x^2

$+2$

$\boxed{(2x^2+3)(x^2+2)}$

ex: Factor completely.

no gcf

e) $16x^4 - 24x^2 + 9$ (double) (perfect square trinomial)

$(4x^2 - 3)(4x^2 - 3)$ or $(4x^2 - 3)^2$

$16x^4 - 12x^2 - 12x^2 + 9$ ✓

f) $2x^{11} - 9x^6 + 10x$

gcf

$x(2x^{10} - 9x^5 + 10)$ (double)

$x(x^5 - 2)(2x^5 - 5)$

$+20$

$\frac{2x^5}{-4}$ $\frac{2x^5}{-5}$

$\div 2$

$\frac{x^5}{-2}$

ex: Consider the following functions...

$a(x) = -2x$ $\qquad$ $b(x) = x^2 - 3x - 5$

$c(x) = 3x^4 + 5x^2$ $\qquad$ $d(x) = 2x^2 - x$

Multiply then simplify.

a) a(x)b(x)

$(-2x)(x^2 - 3x - 5)$

$-2x^3 + 6x^2 + 10x$

Cubic trinomial

ex: Consider the following functions...

$a(x) = -2x$ $\quad$ $b(x) = x^2 - 3x - 5$

$c(x) = 3x^4 + 5x^2$ $\quad$ $d(x) = 2x^2 - x$

Multiply then simplify.

b) $b(x) + a(x)d(x)$ (add, mult.)

$(x^2-3x-5) + (-2x)(2x^2-x)$ (mult.)

$(x^2-3x-5) + (-4x^3+2x^2)$

$x^2-3x-5 + (-4x^3)+2x^2$

$\boxed{-4x^3+3x^2-3x-5}$

Cubic polynomial w/ 4 terms

ex: Consider the following functions...

$$a(x) = -2x \qquad b(x) = x^2 - 3x - 5$$

$$c(x) = 3x^4 + 5x^2 \qquad d(x) = 2x^2 - x$$

Multiply then simplify.

c) a(x)c(x)

$(-2x)(3x^4+5x^2)$

$\boxed{-6x^5-10x^3}$

Quintic binomial

ex: Consider the following functions...

$a(x) = -2x$ $\quad$ $b(x) = x^2 - 3x - 5$

$c(x) = 3x^4 + 5x^2$ $\quad$ $d(x) = 2x^2 - x$

Multiply then simplify.

* d) $[c(x)]^2$ ← squared $= c(x) \cdot c(x)$

$(3x^4 + 5x^2)^2$ Rewrite and FOIL!

$(3x^4 + 5x^2)(3x^4 + 5x^2)$

$9x^8 + 15x^6 + 15x^6 + 25x^4$

$\boxed{9x^8 + 30x^6 + 25x^4}$

8th degree
Trinomial

ex: Consider the following functions...

$$a(x) = -2x \qquad b(x) = x^2 - 3x - 5$$

$$c(x) = 3x^4 + 5x^2 \qquad d(x) = 2x^2 - x$$

Multiply then simplify.

* e) b(x)d(x)

• distribute each term

$(x^2-3x-5)(2x^2-x)$

$2x^4 - 1x^3 - 6x^3 + 3x^2 - 10x^2 + 5x$

$$2x^4 - 7x^3 - 7x^2 + 5x$$

Quartic Polynomial w/ 4 terms

ex: Consider the following functions...

$a(x) = -2x$ $\quad$ $b(x) = x^2 - 3x - 5$

$c(x) = 3x^4 + 5x^2$ $\quad$ $d(x) = 2x^2 - x$

Multiply then simplify.

f) b(x)c(x)

$(x^2 - 3x - 5)(3x^4 + 5x^2)$

$3x^6 + 5x^4 - 9x^5 - 15x^3 - 15x^4 - 25x^2$

$\boxed{3x^6 - 9x^5 - 10x^4 - 15x^3 - 25x^2}$

6th degree Polynomial w/ 5 terms

MIXED PRACTICE

ex: Factor completely.

a) $\frac{3x^3}{3} - \frac{3}{3}$

GCF: $3(x^3-1) = \boxed{3(x-1)(x^2+x+1)}$

SOAP

$a = x \quad a^2 = x^2$

$b = 1 \quad ab = x$

$b^2 = 1$

b) $9x^2+1 = \boxed{\text{Prime}}$

no gcf

↑ sum of squares

ex: Factor completely.

c) $\frac{2x^4}{2} + \frac{8x^2}{2} - \frac{10}{2}$

GCF: $2(x^4 + 4x^2 - 5)$ (double)

-5

$\frac{x^2}{-1}$ $\frac{x^2}{+5}$

$2(x^2-1)(x^2+5)$

DOS

$\boxed{2(x+1)(x-1)(x^2+5)}$

d) $8x^3 - 1$

$= \boxed{(2x-1)(4x^2+2x+1)}$

5 0 AP

no gcf

$a = 2x \quad a^2 = 2x \cdot 2x = 4x^2$

$b = 1 \quad ab = 2x$

$b^2 = 1$