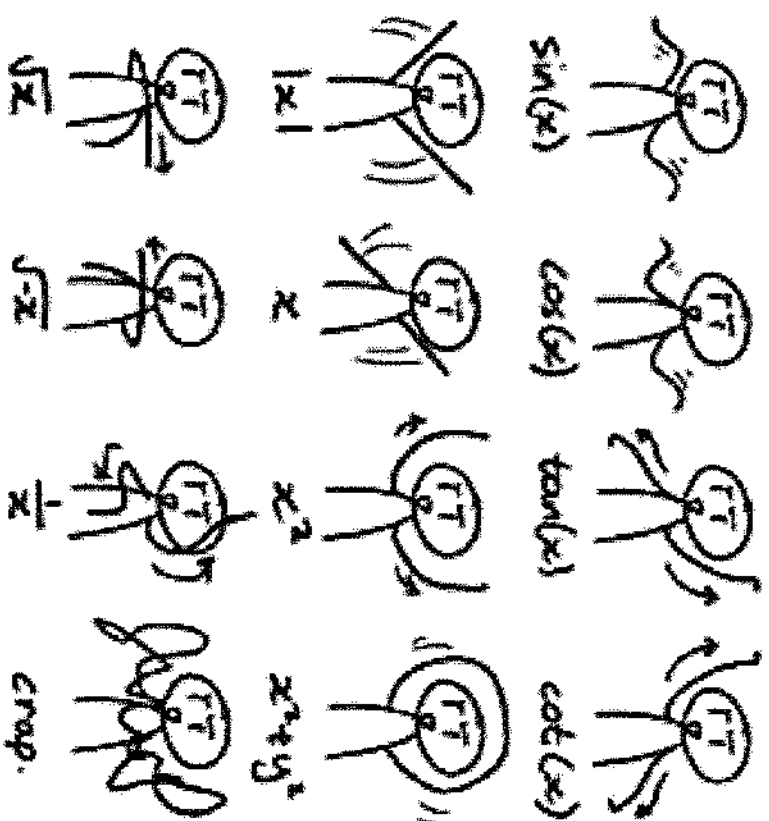


Beautiful Dance Moves



Function Transformations

$$y = af(b(x-h)) + k$$

Types of Transformations

- Shifts (vertical and horizontal)
- Dilations (vertical and horizontal)
- Reflections (about the x-axis, y-axis and origin)

Shifts

$$y = af(b(x-h)) + k$$

Vertical Shifts

Consider: k

$k > 0$ Up

$k < 0$ Down

Horizontal Shifts

Consider: h

$h > 0$ right

$h < 0$ left

- To find h , take opposite sign

Dilations

$$y = af(b(x-h)) + k$$

Vertical

Consider: $\underline{a} \rightarrow$ leading coefficient

$|a| > 1$ Stretch

$|a| < 1$ shrink

Horizontal

Consider: $\underline{b} \rightarrow$ coefficient of \underline{x}

$|b| > 1$ shrink

$|b| < 1$ Stretch

• opposite
of
expectation

Reflections

$$y = af(b(x-h)) + k$$

About the x-axis

$$a < 0$$

About the y-axis

$$b < 0$$

About the origin

$$a < 0 \text{ \& } b < 0$$

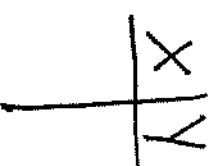
(reflects over
both the x and y-axis)

Sketching Graphs with "Key Points"

- Absolute Value
- Quadratic
- Square Root
 - Cubic
- Cube Root

Process

1. Plot the key point. (h, k)
2. Make a table of values.



ex: Sketch and state the D/R.

Cube Root

$$a) y = -\sqrt[3]{x-4} + 1$$



$a = -1$ • reflection over x -axis

$$b = 1$$

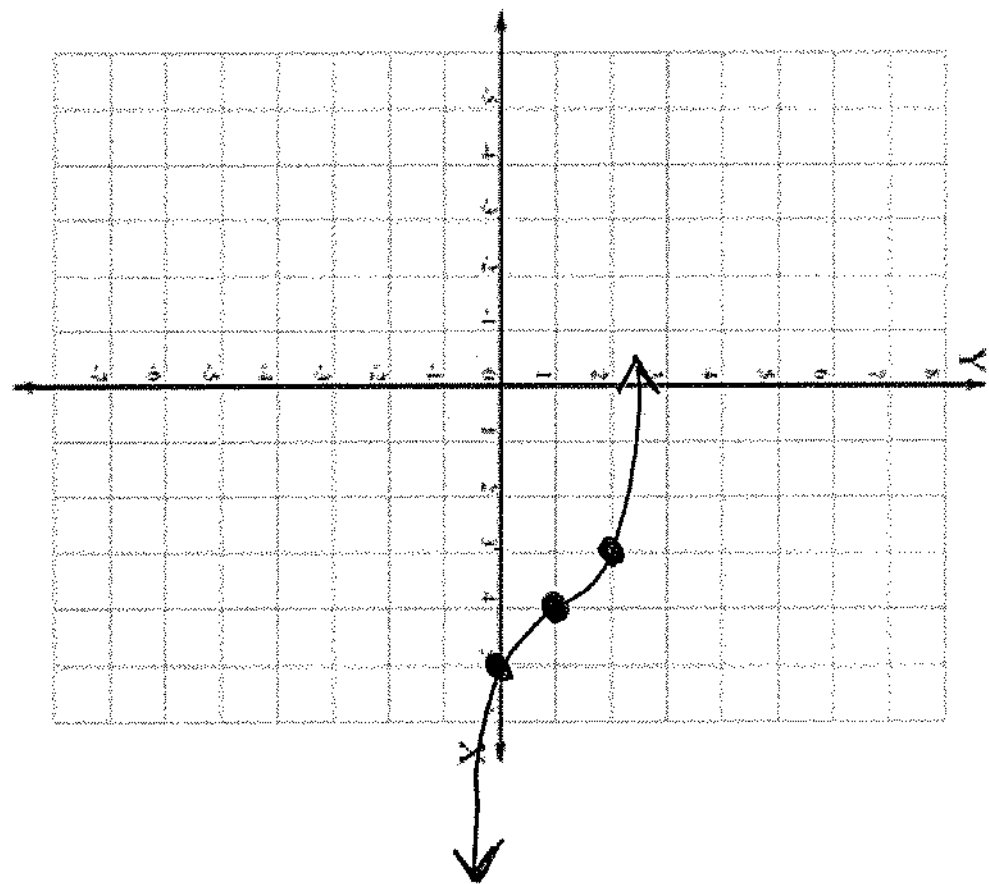
$h = 4$ • right + 4

$k = 1$ • up 1

(h, k)

$(4, 1)$

x	y
3	2
4	1
5	0



SET:

Domain:

$$\{x | x \in \mathbb{R}\}$$

Range:

$$\{y | y \in \mathbb{R}\}$$

ex: Sketch and state the D/R.

$$d) y = 2(x - 4)^3 + 3$$

Cubic



$a = 2$ • Vertical stretch by a factor of 2

$$b = 1$$

$h = 4$ • right 4

$k = 3$ • up 3

(h, k)

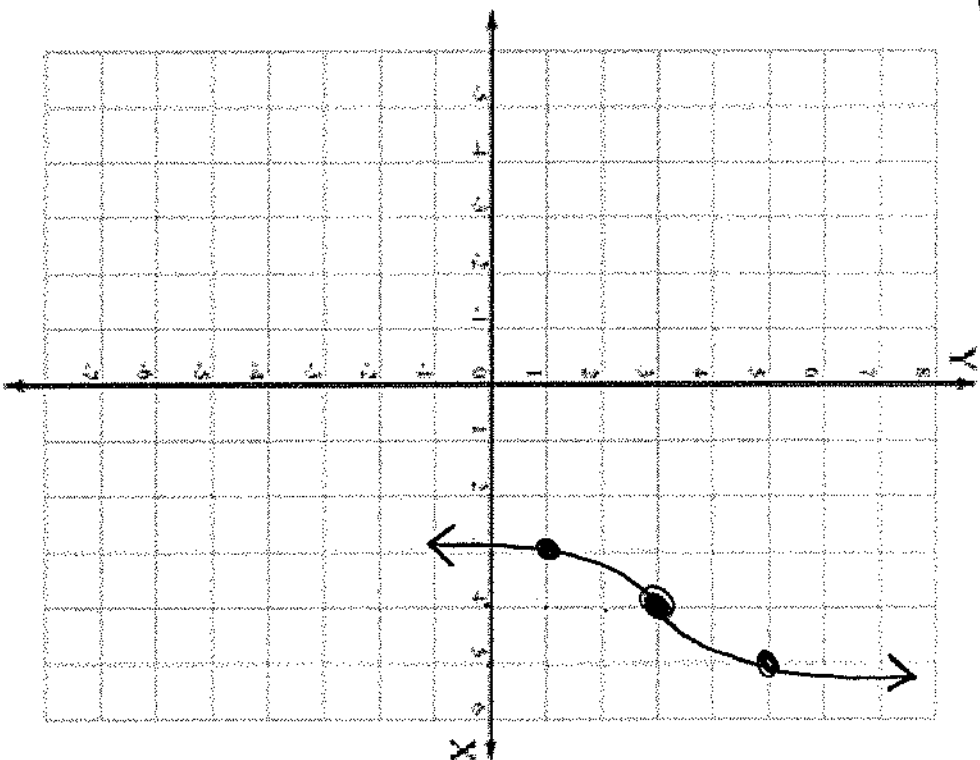
$(4, 3)$

*

X	Y
3	1
4	3
5	5

INTERVAL:

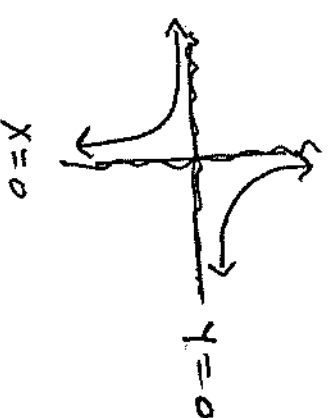
Domain:	Range:
$(-\infty, \infty)$	$(-\infty, \infty)$



Sketching Graphs with Asymptotes

$$y = \frac{1}{x}$$

- Reciprocal



Process

1. Find the asymptote(s).
2. Plot the key point.
3. Make a table of values.

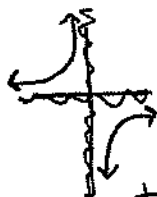
ex: Sketch and state the D/R.

Domain: $\{x \mid x \neq -3\}$

Range: $\{y \mid y \neq 4\}$

a) $y = \frac{2}{x+3} + 4$

Parent: Reciprocal
 $f(x) = \frac{1}{x}$



$$y = 2 \cdot \frac{1}{x+3} + 4$$

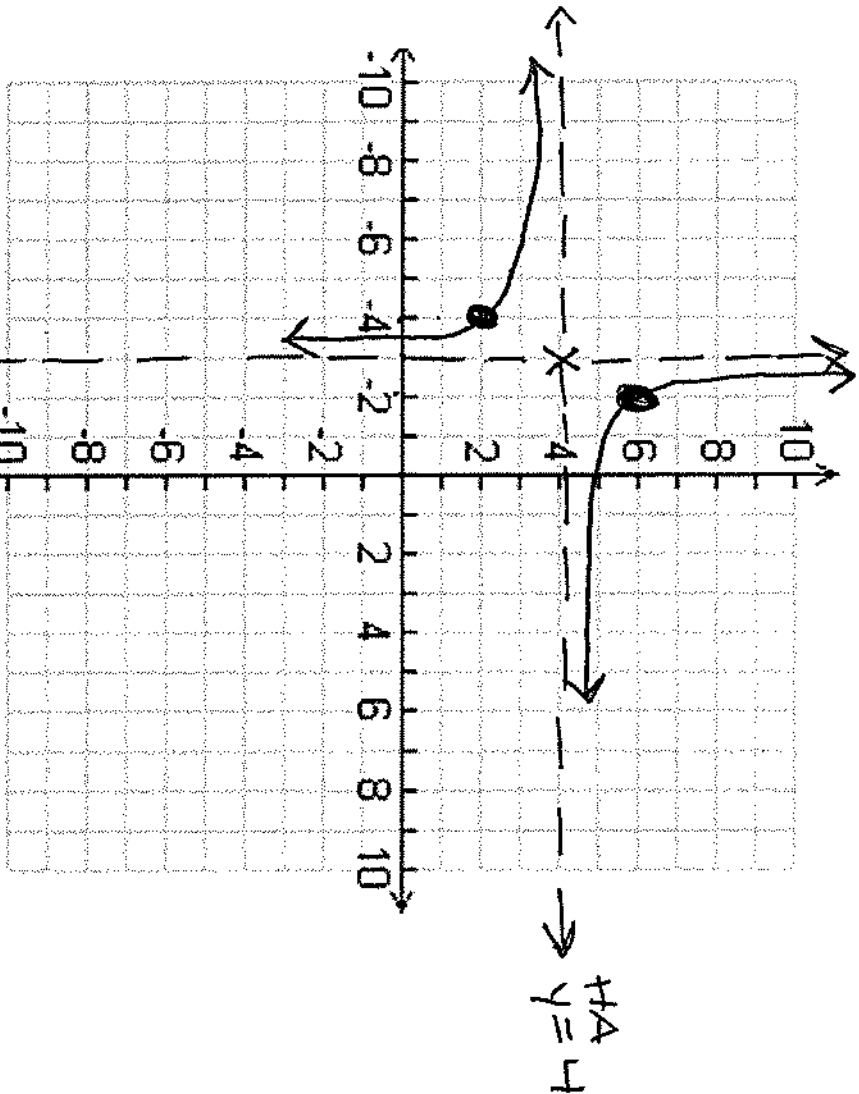
$a = 2$ • Vertical stretch by a factor of 2

$b = 1$

$h = -3$ • Left + 3
 now VA: $x = -3$

$k = 4$ • Up 4
 now HA: $y = 4$

(h, k)
 $(-3, 4)$



★ where the asymptotes cross.

X	Y
-4	2
-3	4
-2	6

$y = \frac{2}{-4+3} + 4$
 $y = \frac{2}{-1} + 4$
 $y = -2 + 3 + 4$
 $y = 6$

ex: Sketch and state the D/R.

Parent: Reciprocal

INTERVAL:
 Domain: $(-\infty, 4) \cup (4, \infty)$
 Range: $(-\infty, 1) \cup (1, \infty)$

b) $y = \frac{-1}{x-4} + 1$

$f(x) = \frac{1}{x}$

$a = -1$ • reflection over the x-axis

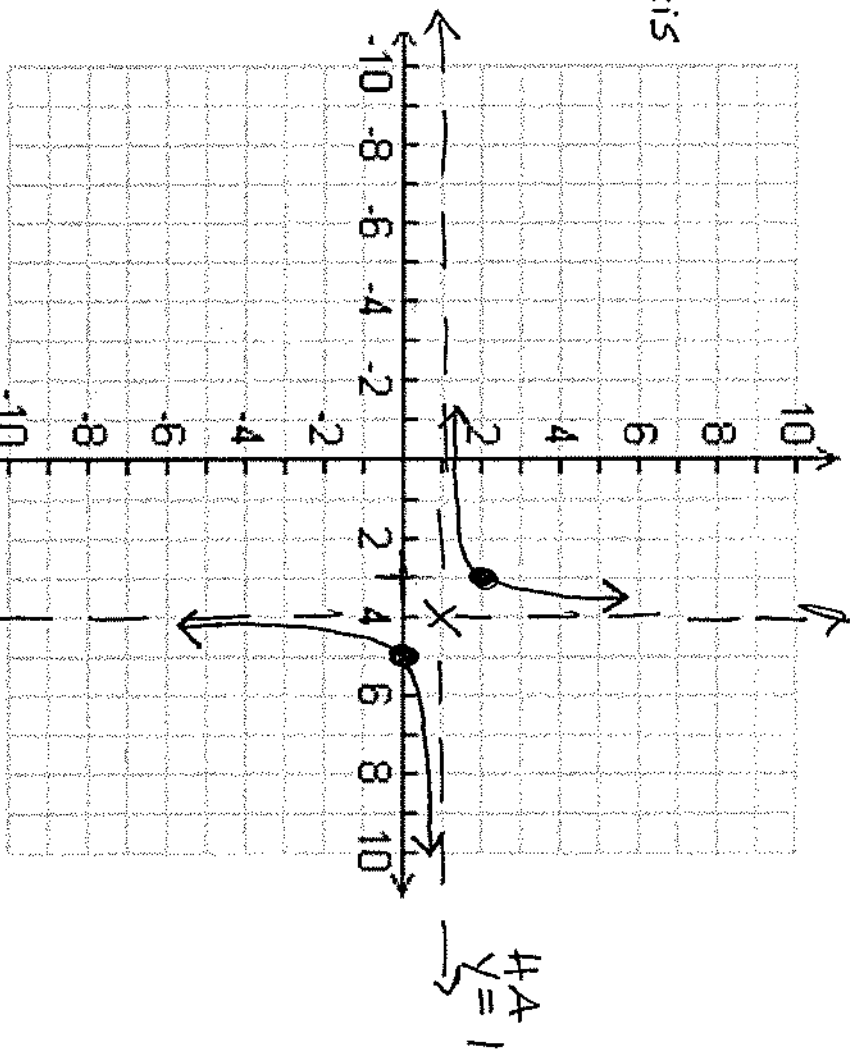
$b = 1$

$h = 4$ • right 4 (VA: $x = 4$)

$k = 1$ • up 1 (HA: $y = 1$)

(h, k)

$(4, 1)$



★

X	Y
3	2
4	1
5	0

where the asymptotes cross

$y = \frac{-1}{x-4} + 1$
 $y = \frac{-1}{1-4} + 1$
 $y = -1 + 1 = 0$

 $X = 4$ VA
 $y = \frac{-1}{3-4} + 1$
 $y = 1 + 1 = 2$

Review.

Name the parent function and describe the transformations.

a) $y = \sqrt[3]{x-4} + 1$

Cube Root

$a = 1$
 $b = 1$
 $h = 4$
 $k = 1$

• right 4; up 1

b) $y = 3(x+2)^2 + 5$

Quadratic

$a = 3$
 $b = 1$
 $h = -2$
 $k = 5$

• Vertical stretch by a factor of 3;
left 2; up 5

c) $y = \frac{1}{3}\sqrt{x-2} - 5$

Square Root

$a = \frac{1}{3}$
 $b = 1$
 $h = 2$
 $k = -5$

• Vertical shrink by a factor of 3;
right 2; down 5

★ d) $y = \sqrt{1-x} + 1$

Square Root

$y = \sqrt{-x+1} + 1$

$y = \sqrt{-(x-1)} + 1$

★ factor out b

$a = 1$
 $b = -1$
 $h = 1$
 $k = 1$

• y-axis reflection; right 1;
up 1