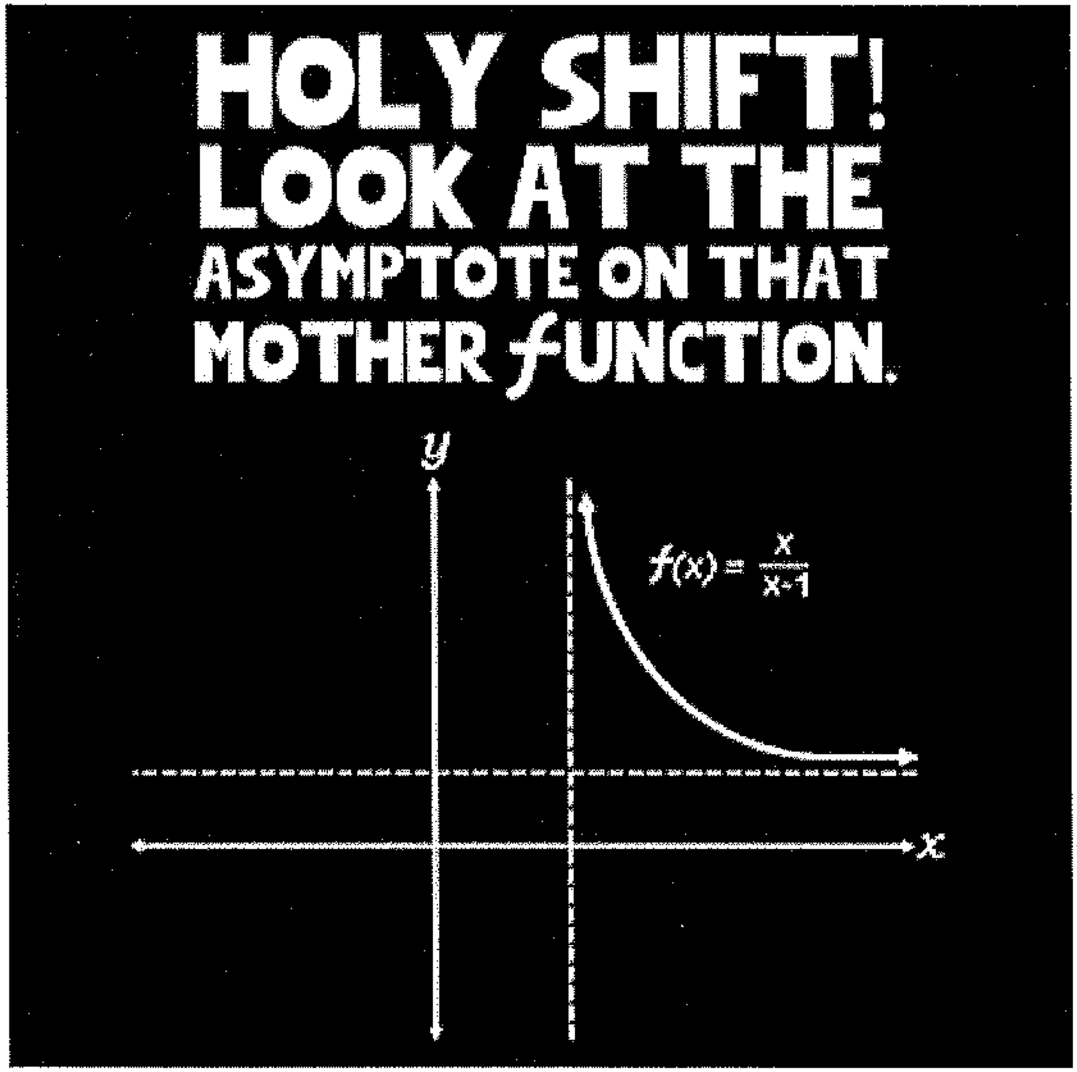
Library of Functions & Transformations - Day 2

Notes

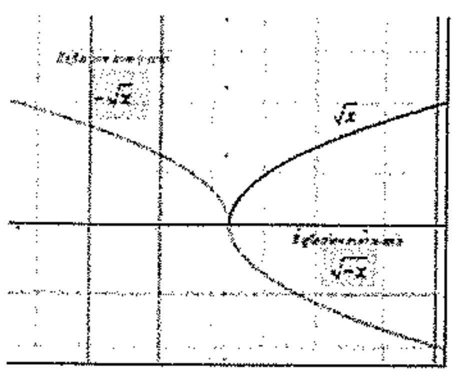


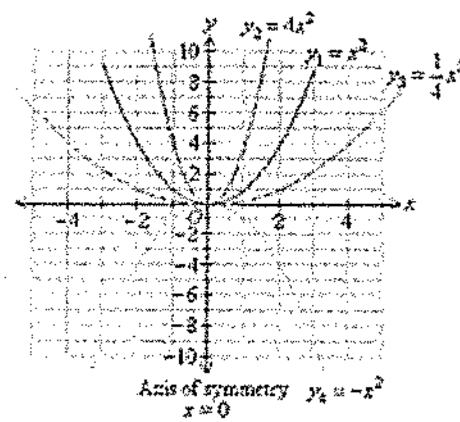
Function Transformations

$$\boxed{y = af\left(b\left(x-h\right)\right)+k}$$

Types of Transformations

- Shifts (vertical and horizontal)
- Dilations (vertical and horizontal)
- Reflections (about the x-axis, y-axis and origin)





Review: Identify the parent function. Then, describe the transformations from the parent function.

a) $f(x) = |x - 1| + 3$

$h = 1$ $\quad$ $k = 3$

absolute value

$y = |x|$

- Right 1
- Up 3

Review: Identify the parent function. Then, describe the transformations from the parent function.

b) $f(x) = (x + 1)^2$

$h = -1$ $K = 0$

Quadratic

$y = x^2$

- Left 1

Review: Identify the parent function. Then, describe the transformations from the parent function.

c) $f(x) = 3 + [\![x + 7]\!]$

Greatest Integer

$y = [\![x]\!]$

$y = [x]$

$y = \text{int}(x)$

$f(x) = [\![x+7]\!] + 3$

$h = -7$ $k = 3$

- Left 7
- Up 3

Review: Identify the parent function. Then, describe the transformations from the parent function.

d) $f(x) = \sqrt[3]{x-2} + 1$

$h=2$ $\quad$ $K=1$

Cube root

$y = \sqrt[3]{x}$

- Right 2
- Up 1

Dilations

$$\boxed{y = af\left(b\left(x-h\right)\right)+k}$$

Vertical

★ Consider: a → leading coefficient

$|a| > 1$ stretch

$|a| < 1$ shrink

Horizontal

★ Consider: b → coefficient of x

Careful

$|b| > 1$ shrink

$|b| < 1$ stretch

ex: Transform the given function f(x) as described and write the resulting function as an equation.

a) $f(x) = [\![x]\!]$

$k=2$ $h=6$

- translated up 2; right 6
- expand vertically by a factor of 4

stretch $a=4$

$$\boxed{f(x) = 4[\![x-6]\!]+2}$$

ex: Transform the given function f(x) as described and write the resulting function as an equation.

b) $f(x) = x^2$

- translated down 8 $K = -8$
- compress horizontally by factor of 2

shrink

$b = 2$

$$\boxed{f(x) = (2x)^2 - 8}$$

b is inside the parent function

not

$2x^2 - 8$

$a = 2$

or

$2(x)^2 - 8$

$a = 2$

ex: Describe the transformations necessary to transform the graph of f(x) into that of g(x).

a) $f(x) = \sqrt{x}$

$g(x) = \sqrt{5(x-1)} - 2$

$b=5$ • horz. shrink by a factor of 5

$h=1$ • right 1

$k=-2$ • Down 2

ex: Describe the transformations necessary to transform the graph of f(x) into that of g(x).

b) $f(x) = |x|$

$$g(x) = \frac{1}{3}|x - 1| + 3$$

$a = \frac{1}{3}$ • Vertical shrink by a factor of 3

$h = 1$ • Right 1

$k = 3$ • Up 3

ex: Describe the transformations necessary to transform the graph of f(x) into that of g(x).

c) $f(x) = \sqrt[3]{x}$

$$g(x) = \sqrt[3]{\frac{1}{4}x - 2}$$

$b = \frac{1}{4}$ • horizontal stretch by a factor of 4

$h = 0$

$k = -2$ • Down 2

ex: Describe the transformations necessary to transform the graph of f(x) into that of g(x).

d) $f(x) = [x]$

$$g(x) = 3\left[\frac{1}{8}(x+2)\right]$$

$a = 3$ • Vertical stretch by a factor of 3

$b = \frac{1}{8}$ • Horizontal stretch by a factor of 8

$h = -2$ • Left 2

$k = 0$

Reflections

$$\boxed{y = af\left(b\left(x-h\right)\right)+k}$$

over
about
in

<u>About the x-axis</u>

$a<0$

<u>About the y-axis</u>

$b<0$

<u>About the origin</u>

$a<0$ & $b<0$

which means the graph reflected over both the x-axis and y-axis

ex: Describe the transformations necessary to transform the graph of f(x) into that of g(x).

a) $f(x) = \sqrt{x}$

$g(x) = -\sqrt{3(x-5)}$

$a = -1$ • reflection over (about / in) the x-axis

$b = 3$ • horizontal shrink by a factor of 3

$h = 5$ • Right 5

$k = 0$

ex: Describe the transformations necessary to transform the graph of f(x) into that of g(x).

b) $f(x) = \sqrt[3]{x}$

$$g(x) = \sqrt[3]{-(x+3)} - 1$$

$a = 1$

$b = -1$ • reflection about over in the y-axis

$h = -3$ • Left 3

$K = -1$ • Down 1

ex: Describe the transformations necessary to transform the graph of f(x) into that of g(x).

c) $f(x) = [x]$

$g(x) = -[-2(x+1)]$

$a = -1$ • reflection about the origin (both x & y-axis)

$b = -2$ • horizontal shrink by a factor of 2

$|b| > 1$

$h = -1$ • Left 1

$K = 0$

Sketching Graphs with "Key Points"

- Absolute Value
- Quadratic
- Square Root
- Cubic
- Cube Root

Process

1. Plot the key point. (h, k)
2. Make a table of values.

ex: Describe the transformations from the parent function then sketch the function. State the D/R in any notation.

Quadratic (U-shaped)

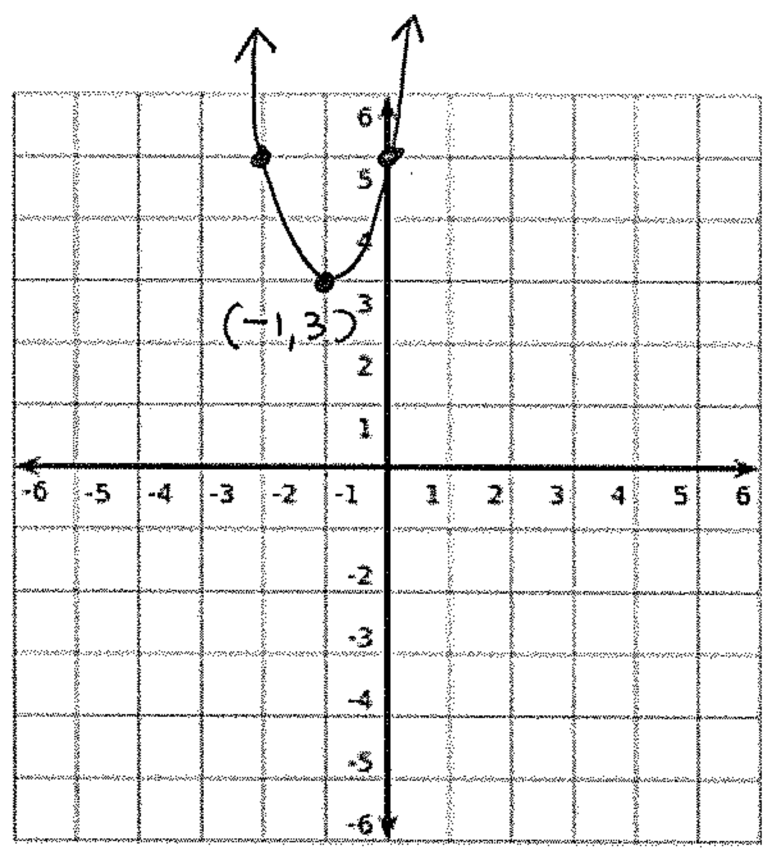
a) $f(x) = 2(x+1)^2 + 3$

$a = 2$
$b = 1$
$h = -1$
$k = 3$

- Vertical stretch by a factor of 2
- Left 1
- Up 3

$(-1, 3)$
(h, k)

x	y
-2	5
* -1	3
0	5



Set:

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq 3\}$

ex: Describe the transformations from the parent function then sketch the function. State the D/R in any notation.

absolute value (V-shaped)

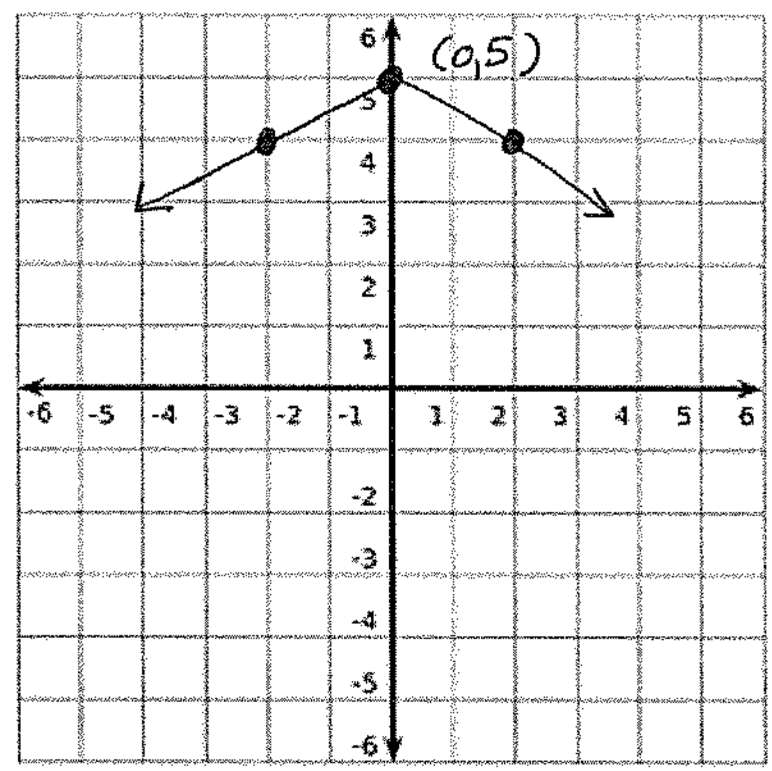
b) $f(x) = 5 - \frac{1}{2}|x|$

$f(x) = -\frac{1}{2}|x| + 5$

$a = -\frac{1}{2}$ • reflection over the x-axis
$b = 1$ • vertical shrink by a factor of 2
$h = 0$

$K = 5$ • up 5

(h, K)
$(0, 5)$



x	y
-2	4
★ 0	5
2	4

Works best with the fraction, divisible by 2.

Interval:

D: $(-\infty, \infty)$

R: $(-\infty, 5]$

ex: Describe the transformations from the parent function then sketch the function. State the D/R in any notation.

square root

c) $f(x) = \sqrt{-3(x-2)}$

$a = 1$

$b = -3$ • reflection over the y-axis

• horizontal shrink by a factor of 3

$h = 2$

$k = 0$ • Right 2

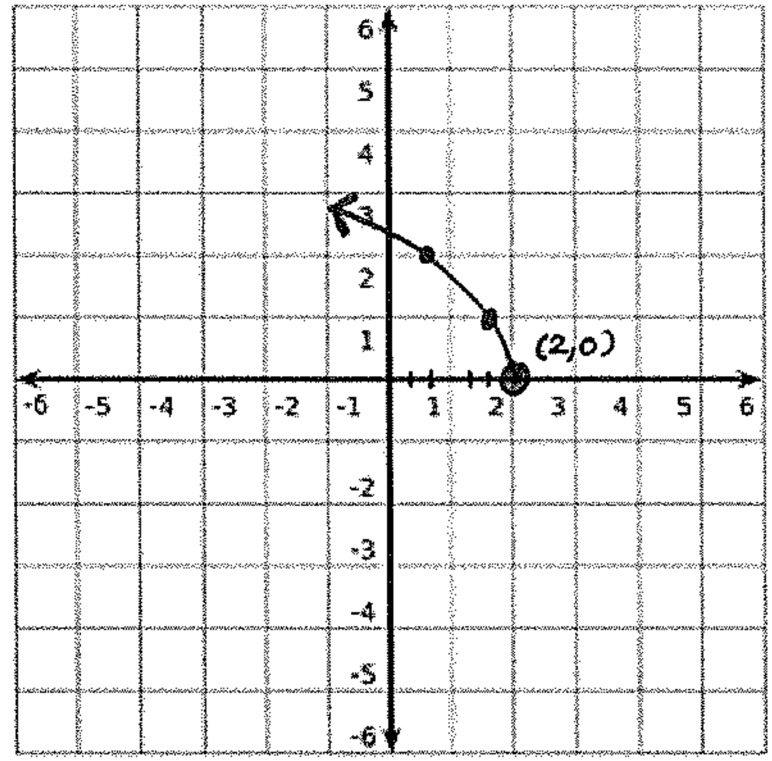
(h, k)

$(2, 0)$

Interval:

D: $(-\infty, 2]$

R: $[0, \infty)$



x	y
★ 2	0
$1\frac{2}{3} \leftarrow \frac{5}{3}$	1
$\frac{2}{3}$	2

• need to create perfect squares

$-3(x-2) = 1$

$-3x + 6 = 1$

$\quad -6 \quad -6$

$-3x = -5$

$x = \frac{5}{3}$

$f(\frac{5}{3}) = \sqrt{-3(\frac{5}{3}-2)}$

$= \sqrt{-5+6}$

$= \sqrt{1}$

$= 1$

$-3(x-2) = 4$

$-3x + 6 = 4$

$\quad -6 \quad -6$

$-3x = -2$

$x = \frac{2}{3}$

$f(\frac{2}{3}) = \sqrt{-3(\frac{2}{3}-2)}$

$= \sqrt{-2+6}$

ex: Describe the transformations from the parent function then sketch the function. State the D/R in any notation.

Quadratic (U-shaped)

d) $f(x) = -\frac{1}{3}(x-2)^2 + 1$

$a = -\frac{1}{3}$
- reflection over the x-axis
- vertical shrink by a factor of 3

$b = 1$

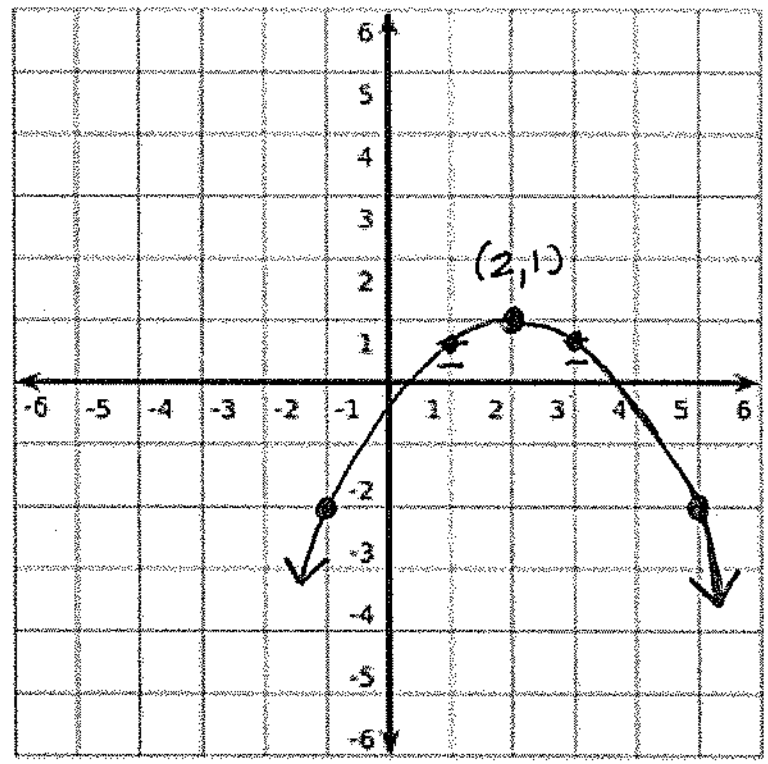
$h = 2$ • Right 2

$K = 1$ • Up 1

(h, K)
$(2, 1)$

X	Y
-1	-2
1	2/3
* 2	1
3	2/3
5	-2

Will create a value that is divisible b...



SET:

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \leq 1\}$

ex: Describe the transformations from the parent function then sketch the function. State the D/R in any notation.

Absolute value (V-shaped)

e) $f(x) = \left|\frac{1}{3}x\right| - 4$

$a=1$

$b=\frac{1}{3}$ • Horizontal stretch by a factor of 3

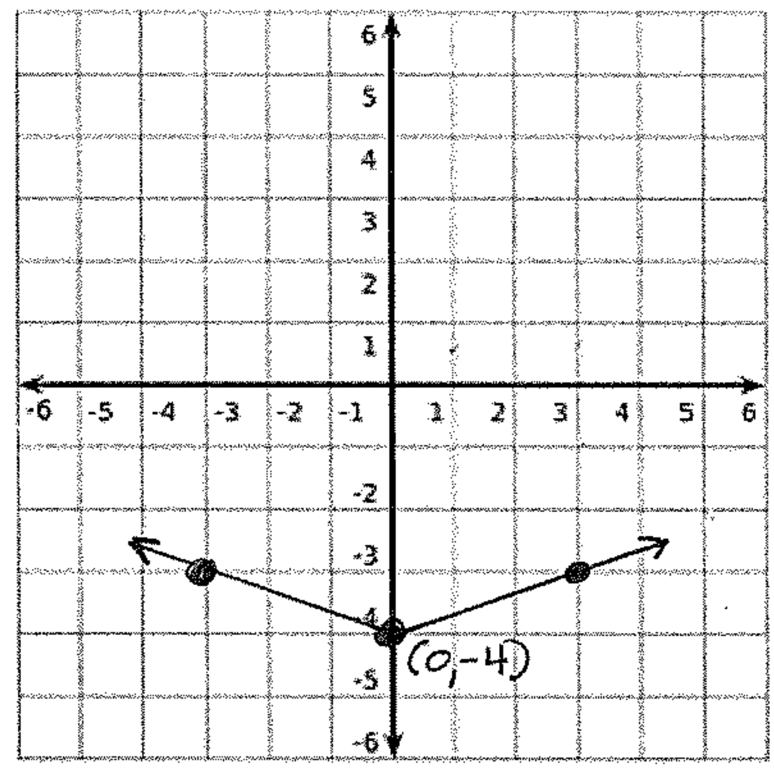
$h=0$

$k=-4$ • Down 4

(h,k)
$(0,-4)$

x	y
-3	-3
* 0	-4
3	-3

will create a value that is divisible by 3.



Interval:

D: $(-\infty,\infty)$

R: $[-4,\infty)$

ex: Describe the transformations from the parent function then sketch the function. State the D/R in any notation.

Square root $\sqrt{}$

f) $f(x) = -\sqrt{-x} + 1$

$a = -1$, $b = -1$ • reflection about the origin (both x & y-axis)

$h = 0$

$k = 1$ • Up 1

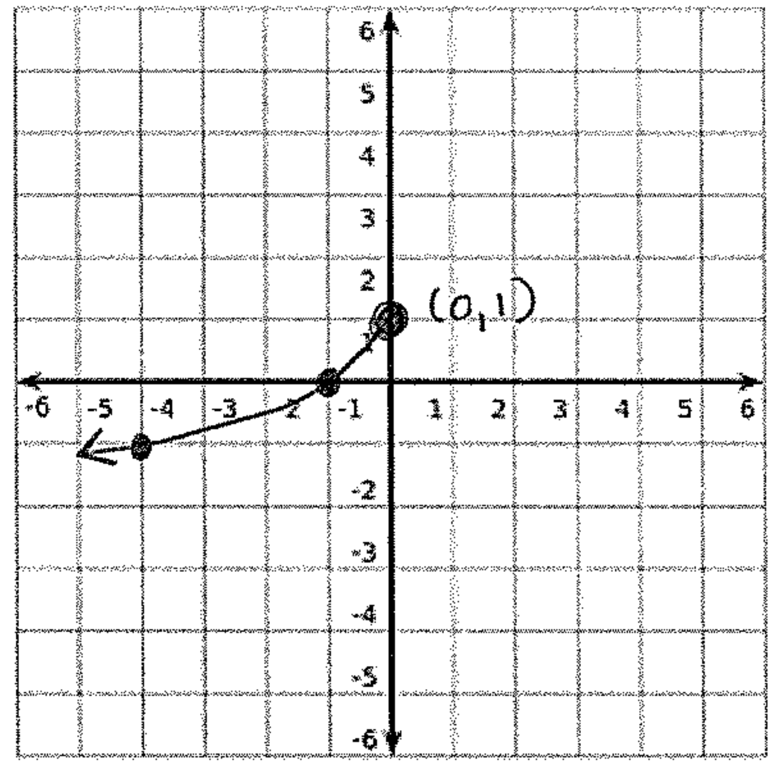
(h, k)
$(0, 1)$

X	Y
* 0	1
−1	0
−4	−1

need perfect squares

$-x = 1$
$x = -1$

$f(-1) = -\sqrt{-(-1)} + 1$
$= -\sqrt{1} + 1$
$= -1 + 1$
$= 0$



$-x = 4$
$x = -4$

$f(-4) = -\sqrt{-(-4)} + 1$
$= -\sqrt{4} + 1$
$= -2 + 1$
$= -1$

Interval:

D: $(-\infty, 0]$

R: $(-\infty, 1]$

ex: Describe the transformations from the parent function then sketch the function. State the D/R in any notation.

square root $\sqrt{x}$

g) $f(x) = 2\sqrt{-(x-3)}$

$a = 2$ • Vertical stretch by a factor of 2

$b = -1$ • reflection over the y-axis

$h = 3$ • Right 3

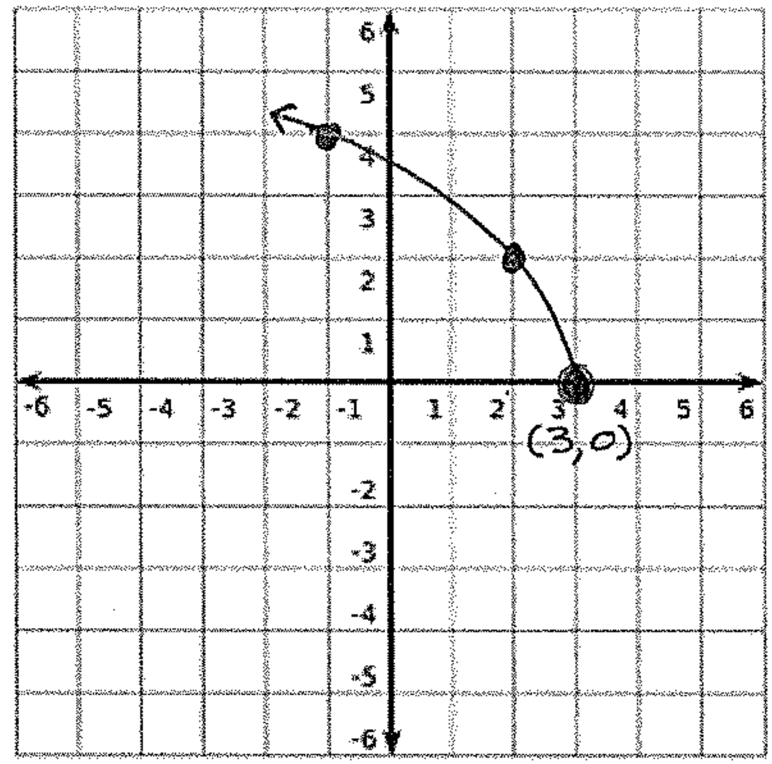
$K = 0$

(h, K)

$(3, 0)$

X	Y
* 3	0
2	2
−1	4

need perfect squares



$-(x-3) = 1$

$-x + 3 = 1$

$\quad -3 \quad -3$

$-x = -2$

$x = 2$

$f(2) = 2\sqrt{-(2-3)}$

$= 2\sqrt{-(-1)}$

$= 2\sqrt{1} = 2$

$-(x-3) = 4$

$-x + 3 = 4$

$\quad -3 \quad -3$

$-x = 1$

$x = -1$

$f(-1) = 2\sqrt{-(-1-3)}$

$= 2\sqrt{-(-4)}$

$= 2\sqrt{4}$

$= 2(2) = 4$

SET:

D: $\{x \mid x \leq 3\}$

R: $\{y \mid y \geq 0\}$