

Library of Functions & Transformations - Day 1

Beautiful Dance Moves



$\sin(x)$



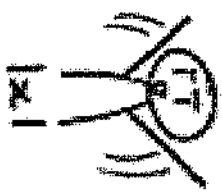
$\cos(x)$



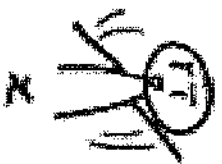
$\tan(x)$



$\cot(x)$



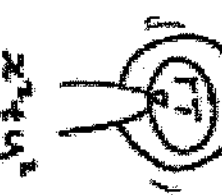
$|x|$



x



x^2



$x^2 + y^2$



\sqrt{x}



\sqrt{x}



$\frac{1}{x}$



$\frac{1}{x}$

Parent Functions (Mother Functions)

A parent function is the simplest function of a family of functions. For the family of quadratic functions,

$y = ax^2 + bx + c$, the simplest function. of this form is $y = x^2$.

In this lesson we will examine several "families" of functions.

A family of functions is a set of functions whose equations have a similar form. The "parent" of the family is the equation in the family with the simplest form. For example,

$y = x^2$ is a parent to other functions, such as $y = 2x^2 - 5x + 3$.

1. Constant

(Horizontal line)

$$f(x) = \underline{\quad C \quad}$$

↙ a number

X	Y
-1	C
0	C
1	C
2	C

D: $\{x | x \in \mathbb{R}\}$

Set

R: $\{y | y = C\}$

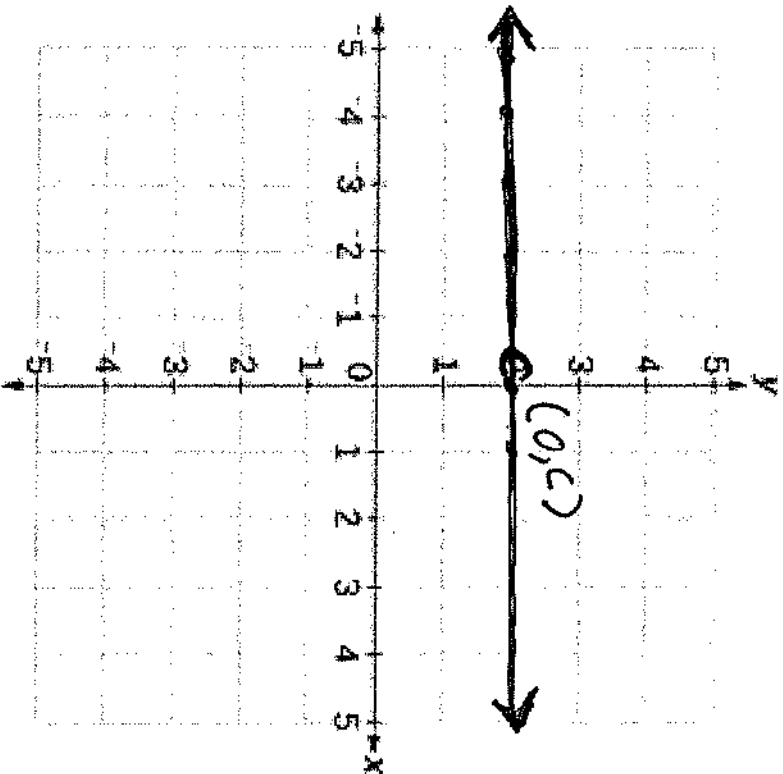
Interval

D: $(-\infty, \infty)$

R: $[C]$ or $[C, C]$

End Behavior : $X \rightarrow -\infty, f(x) \rightarrow C$

(EB) $X \rightarrow \infty, f(x) \rightarrow C$



2. Identity

$$f(x) = \underline{x}$$

x	y
-2	-2
-1	-1
0	0
1	1

$$y = x$$

D: $\{x | x \in \mathbb{R}\}$

Set

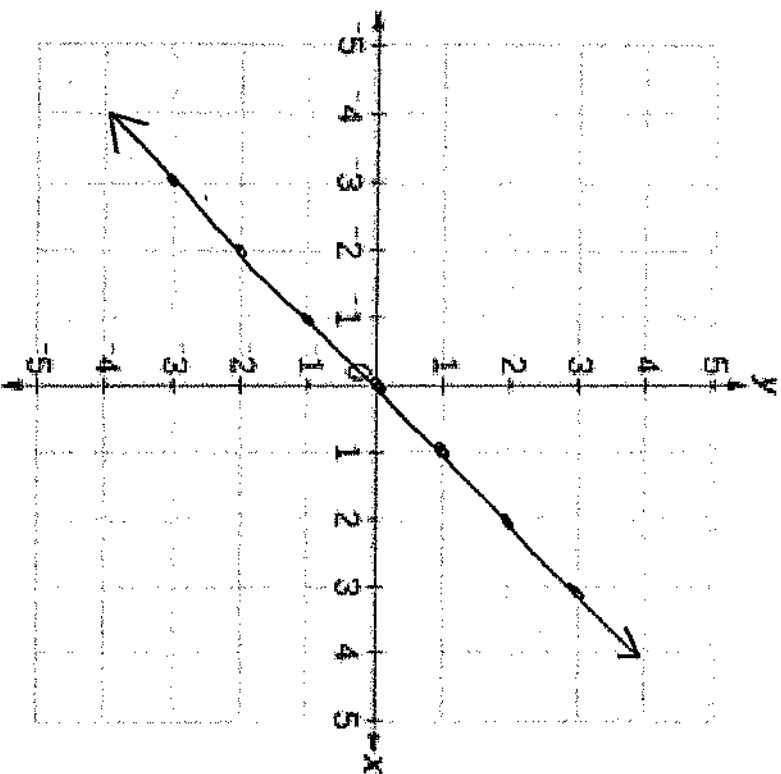
R: $\{y | y \in \mathbb{R}\}$

Interval

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

End Behavior : $x \rightarrow -\infty, f(x) \rightarrow -\infty$
(EB) $x \rightarrow \infty, f(x) \rightarrow \infty$



3. Absolute Value

$$f(x) = \underline{\quad |x| \quad}$$

x	y
-2	2
-1	1
0	0
1	1
2	2

Set

D: $\{x | x \in \mathbb{R}\}$

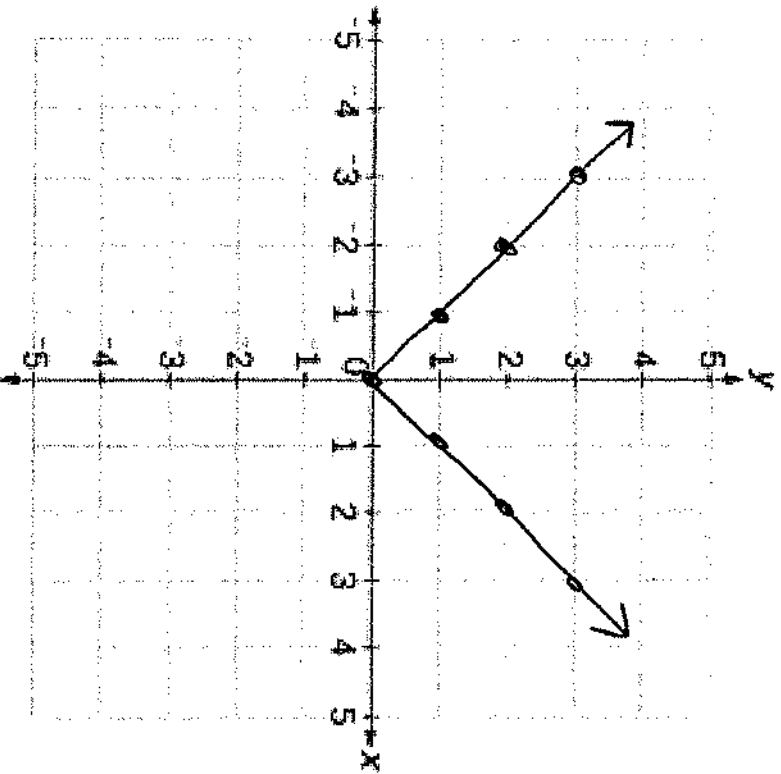
R: $\{y | y \geq 0\}$

Interval

D: $(-\infty, \infty)$

R: $[0, \infty)$

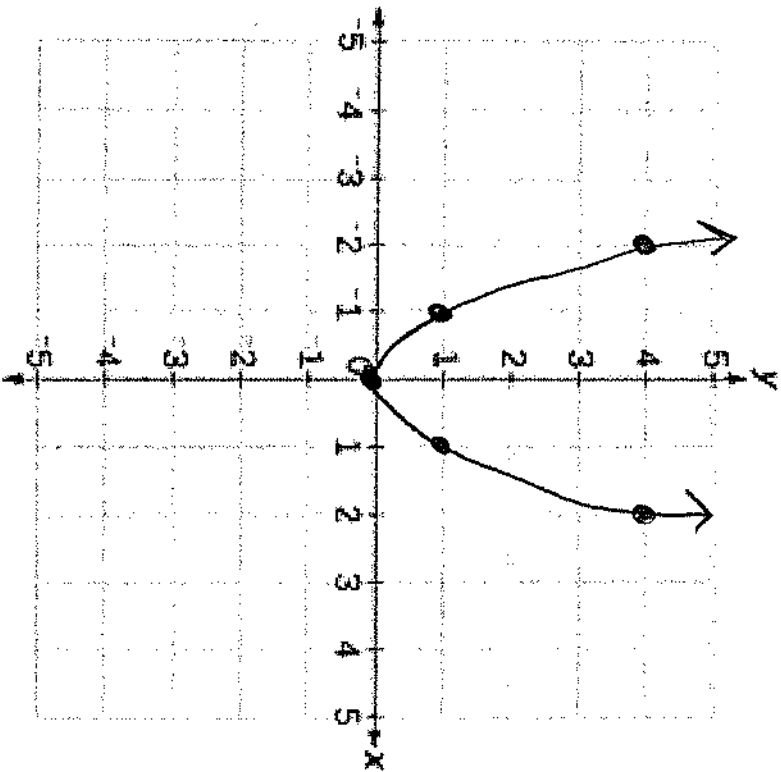
EB : $x \rightarrow -\infty, f(x) \rightarrow \infty$
 $x \rightarrow \infty, f(x) \rightarrow \infty$



4. Quadratic

$$f(x) = \underline{\quad} X^2$$

X	Y
-2	4
-1	1
0	0
1	1
2	4



D: $\{x | x \in \mathbb{R}\}$
Set

R: $\{y | y \geq 0\}$

Interval

D: $(-\infty, \infty)$

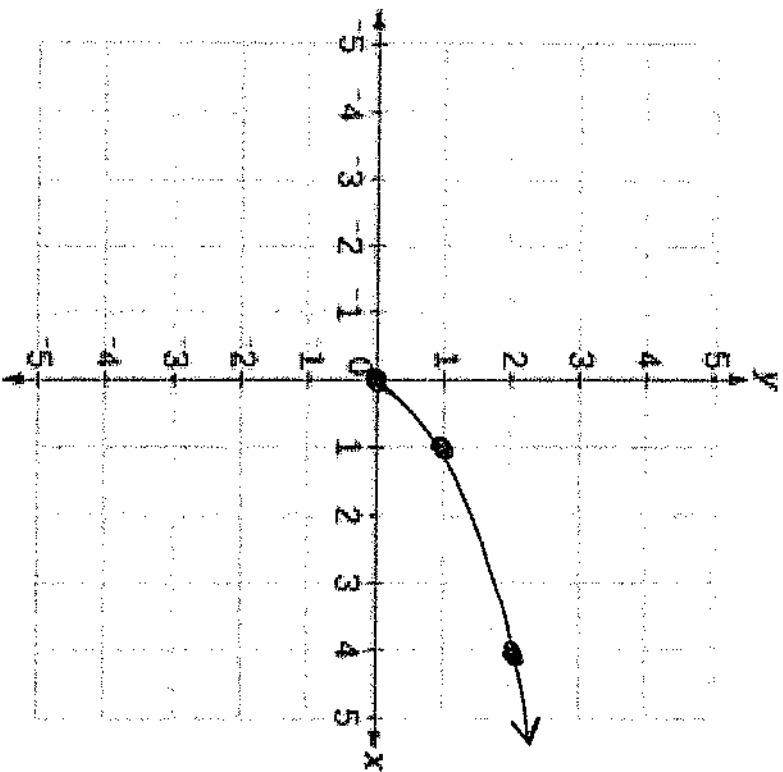
R: $[0, \infty)$

EB : $x \rightarrow -\infty, f(x) \rightarrow \infty$
 $x \rightarrow \infty, f(x) \rightarrow \infty$

5. Square Root

$$f(x) = \sqrt{x}$$

x	y
0	0
1	1
4	2
9	3



D: $\{x | x \geq 0\}$ Set

R: $\{y | y \geq 0\}$

Interval

D: $[0, \infty)$

R: $[0, \infty)$

EB: $x \rightarrow -\infty, f(x) \rightarrow$ Does not exist

$x \rightarrow \infty, f(x) \rightarrow \infty$

6. Reciprocal

HA: $y=0$ (BoBo)

$$\frac{x}{y} \Big| \frac{y}{x}$$

$$f(x) = \frac{1}{x} \quad \leftarrow x \neq 0 \text{ (undefined)}$$

VA: $x=0$

D: $\{x | x \neq 0\}$

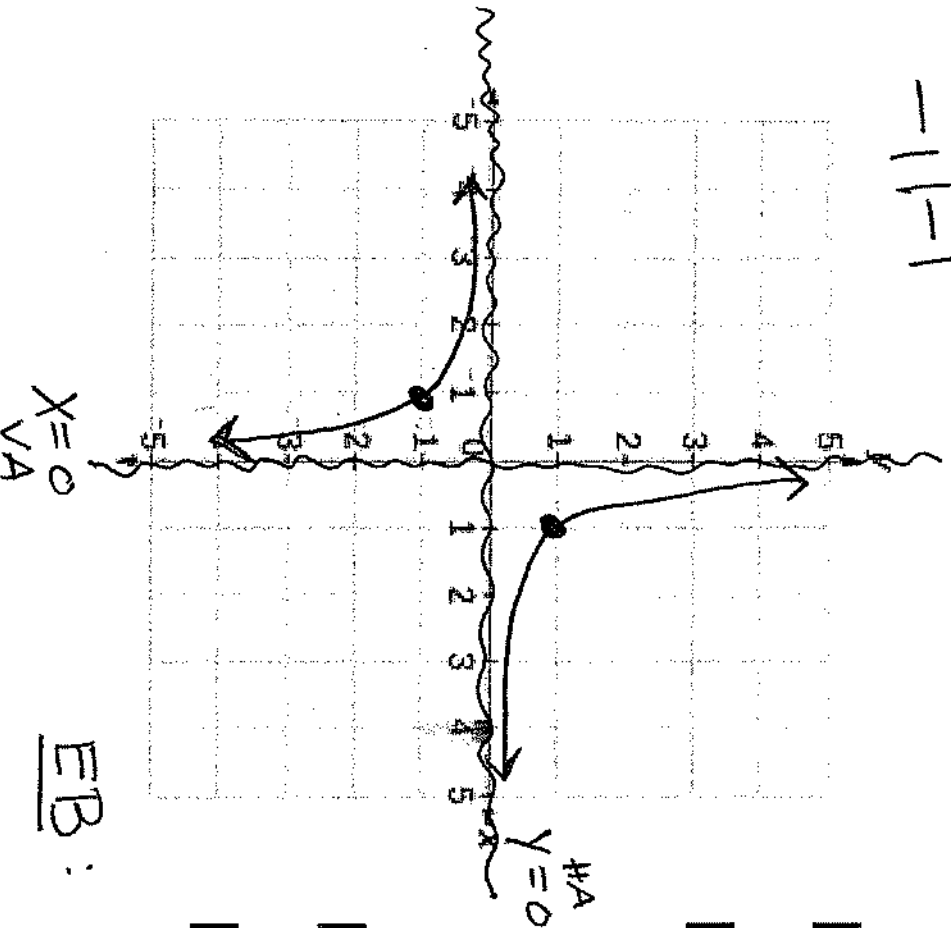
R: $\{y | y \neq 0\}$

Interval

D: $(-\infty, 0) \cup (0, \infty)$

R: $(-\infty, 0) \cup (0, \infty)$

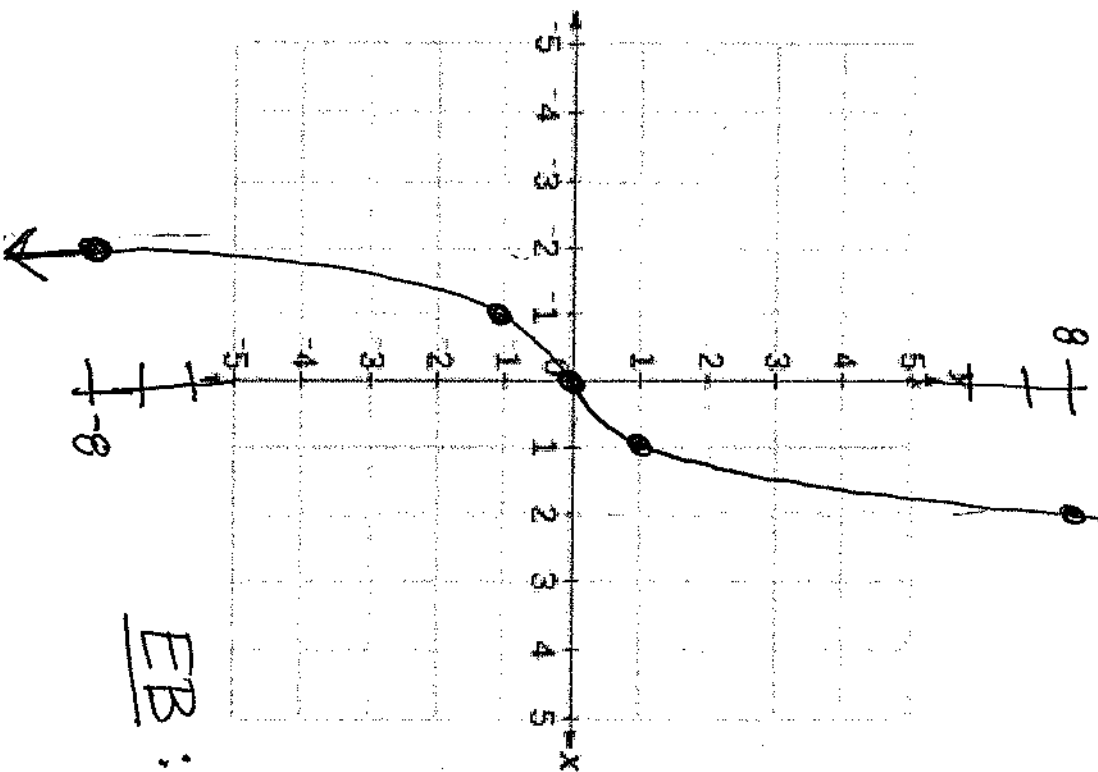
EB: $x \rightarrow -\infty, f(x) \rightarrow 0$ (HA)
 $x \rightarrow \infty, f(x) \rightarrow 0$ (HA)



7. Cubic

$$f(x) = \frac{x^3}{x^2 - 1}$$

x	y
-2	-8
-1	-1
0	0
1	1
2	8



Set

D: $\{x | x \in \mathbb{R}\}$

R: $\{y | y \in \mathbb{R}\}$

Interval

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

$x \rightarrow -\infty, f(x) \rightarrow -\infty$

$x \rightarrow \infty, f(x) \rightarrow \infty$

EB:

8. Cube Root

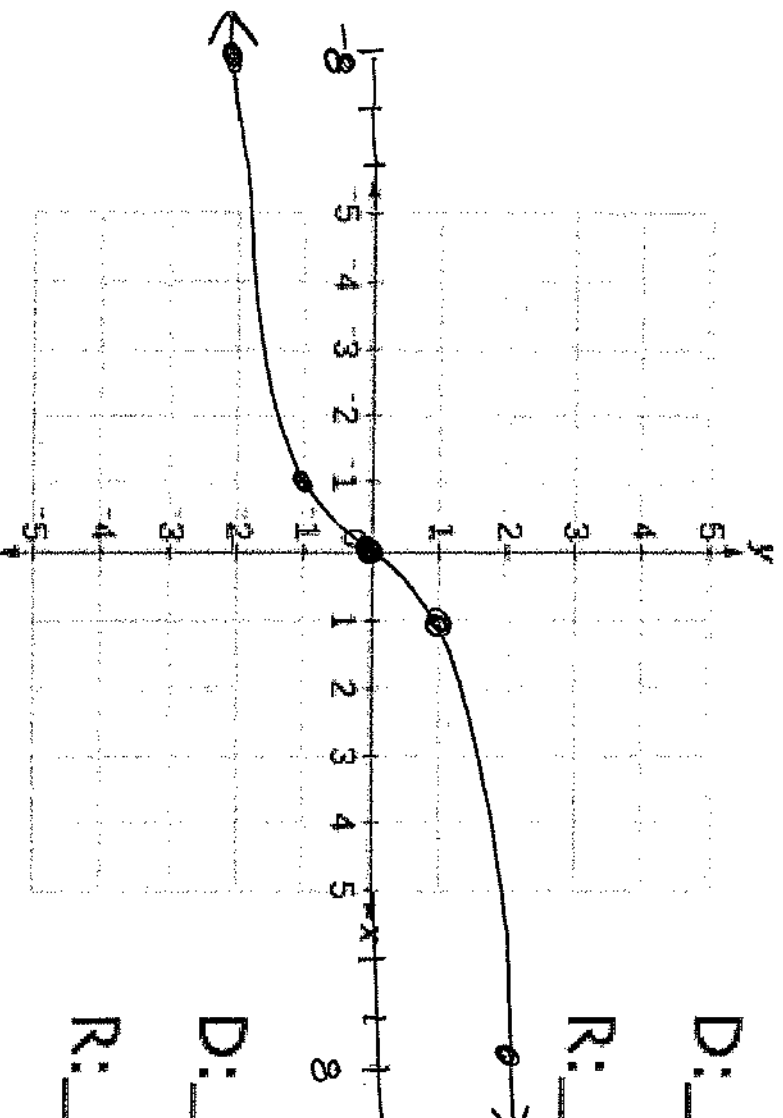
$$f(x) = \sqrt[3]{x}$$

x	y
-8	-2
-1	-1
0	0
1	1
8	2

Set

D: $\{x | x \in \mathbb{R}\}$

R: $\{y | y \in \mathbb{R}\}$



Interval

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

EB:

$x \rightarrow -\infty, f(x) \rightarrow -\infty$

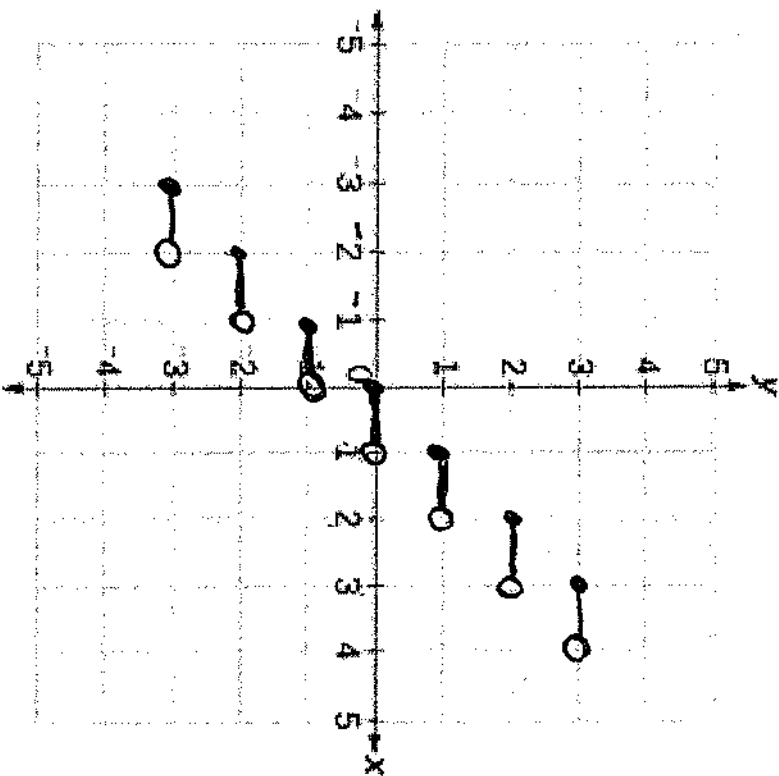
$x \rightarrow \infty, f(x) \rightarrow \infty$

9. Greatest Integer

Integers: (\mathbb{Z})

$\{ \dots, -3, -2, -1, 0, 1, 2, 3, \dots \}$

$$f(x) = \underline{[x]} \text{ or } \underline{\lfloor x \rfloor} \text{ or } \text{int}(x)$$



Set

D: $\{x | x \in \mathbb{R}\}$

R: $\{y | y \in \mathbb{Z}\}$

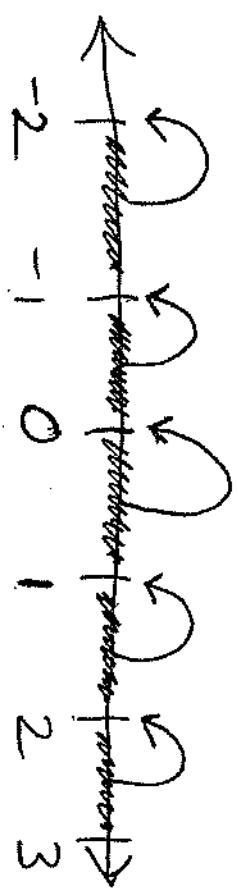
Interval

D: $(-\infty, \infty)$

R: N/A

EB: $x \rightarrow -\infty, f(x) \rightarrow -\infty$

$x \rightarrow \infty, f(x) \rightarrow \infty$



Function Transformations

$$y = af(b(x-h)) + k$$

Types of Transformations

- Shifts (vertical and horizontal)
- Dilations (vertical and horizontal)
- Reflections (about the x-axis, y-axis and origin)

Shifts

$$y = af(b(x - \underline{h})) + \underline{k}$$

Vertical Shifts

Consider: k

$k > 0$ Up

$k < 0$ down

$$(x-2)^2$$

$h=2$



Horizontal Shifts

Consider: h

$h > 0$ Right

$h < 0$ Left

$$(x+2)^2$$

$h=-2$



ex: Identify the parent function. Then, describe the transformations from the parent function.

a) $f(x) = |x + 7| + 4$

$h = -7$
Left 7

Up 4

Absolute value
 $y = |x|$

b) $f(x) = (x - 1)^2$

$h = 1$

+ 0

Right 1

Quadratic
 $y = x^2$

ex: Identify the parent function. Then, describe the transformations from the parent function.

$$c) f(x) = \sqrt[3]{x} - 5$$

Down 5

Cube root
 $y = \sqrt[3]{x}$

$$d) f(x) = 6 + \lceil x - 7 \rceil$$

$$f(x) = \lceil x - 7 \rceil + 6$$

$h = 7$

Up 6
Right 7

Greatest Integer
 $y = \lceil x \rceil$

Describe the transformations necessary to transform the graph of $f(x)$ into the graph of $g(x)$

a) $f(x) = \sqrt{x}$

$$g(x) = -\sqrt{x+4}$$

- reflected over x-axis
- left 4

b) $f(x) = |x|$

$$g(x) = -|x-6| + 1$$

- reflected over x-axis
- right 6
- up 1

Transform the given function $f(x)$ as described and write the resulting function as an equation.

a) $f(x) = x^2$

UP 2; reflected over x-axis

$$g(x) = -x^2 + 2$$

b) $f(x) = \sqrt{x}$

left 2; down 6; reflected over x-axis

* $h = -2$

$$g(x) = -\sqrt{x+2} - 6$$

Sketching ABSOLUTE VALUE Functions

$$f(x) = |x|$$

$$f(x) = a|b(x - h)| + k$$

Process

1. Identify the key point: (h, k)
2. Make a table of values with two other points, one to the left and one to the right of the key point.

ex: Describe the transformations from the parent function then sketch the absolute value function. State the D/R in any notation.

a) $f(x) = |x - 5| - 1$

parent
 $y = |x|$

absolute value
(V-shaped)

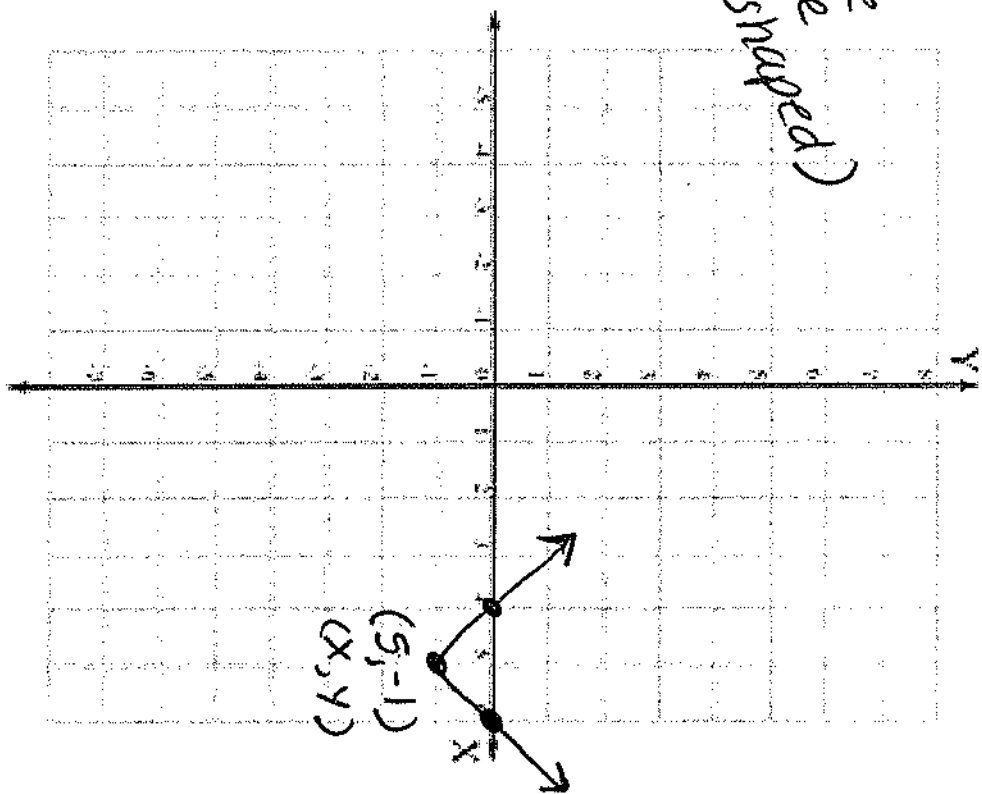
$h = 5$

right 5

down 1

$x - 5 = 0$
 $x = 5$

X	Y
4	0
5	-1
6	0



set

Domain:	Range:
$\{x x \in \mathbb{R}\}$	$\{y y \geq -1\}$

ex: Describe the transformations from the parent function then sketch the absolute value function. State the D/R in any notation.

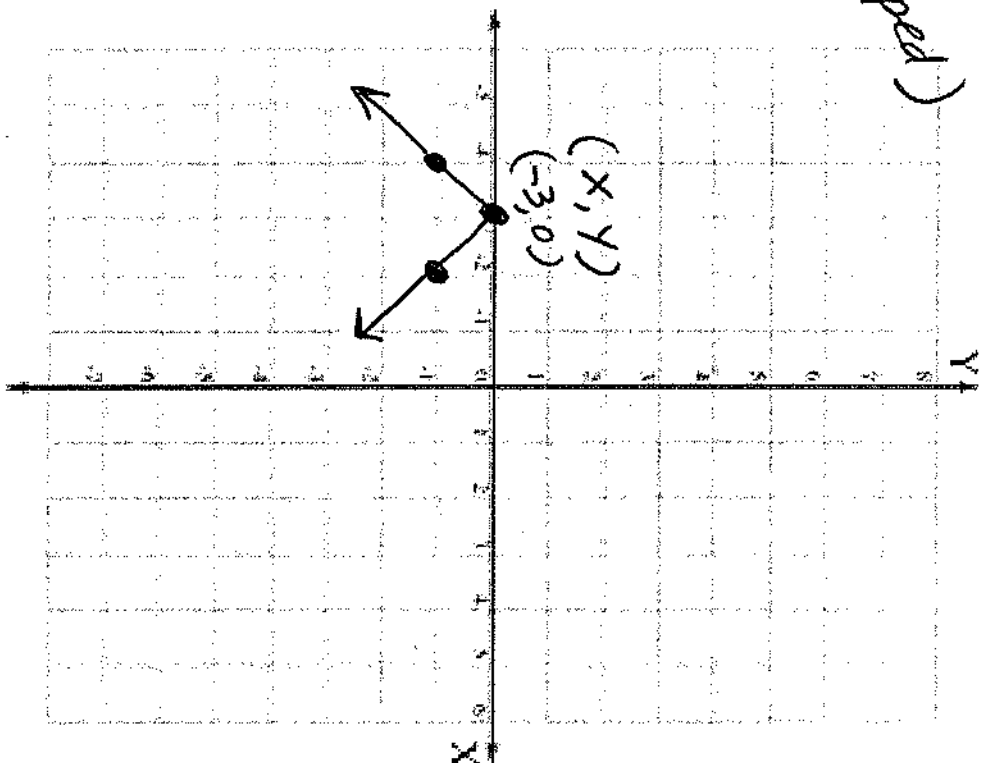
b) $f(x) = -|x + 3|$

absolute value
(v-shaped)

$f(x) = -|x + 3|$ $h = -3$

- reflection over the x-axis
- left + 3
- $x + 3 = 0$
- $x = -3$

X	Y
-4	-1
-3	0
-2	-1



INTERVAL

Domain:	Range:
$(-\infty, \infty)$	$(-\infty, 0]$

Sketching QUADRATIC Functions

$$f(x) = x^2$$

$$f(x) = a(b(x - h))^2 + k$$

Process

1. Identify the key point: (h, k)
2. Make a table of values with two other points, one to the left and one to the right of the key point.

ex: Describe the transformations from the parent function then sketch the quadratic function. State the D/R in any notation.

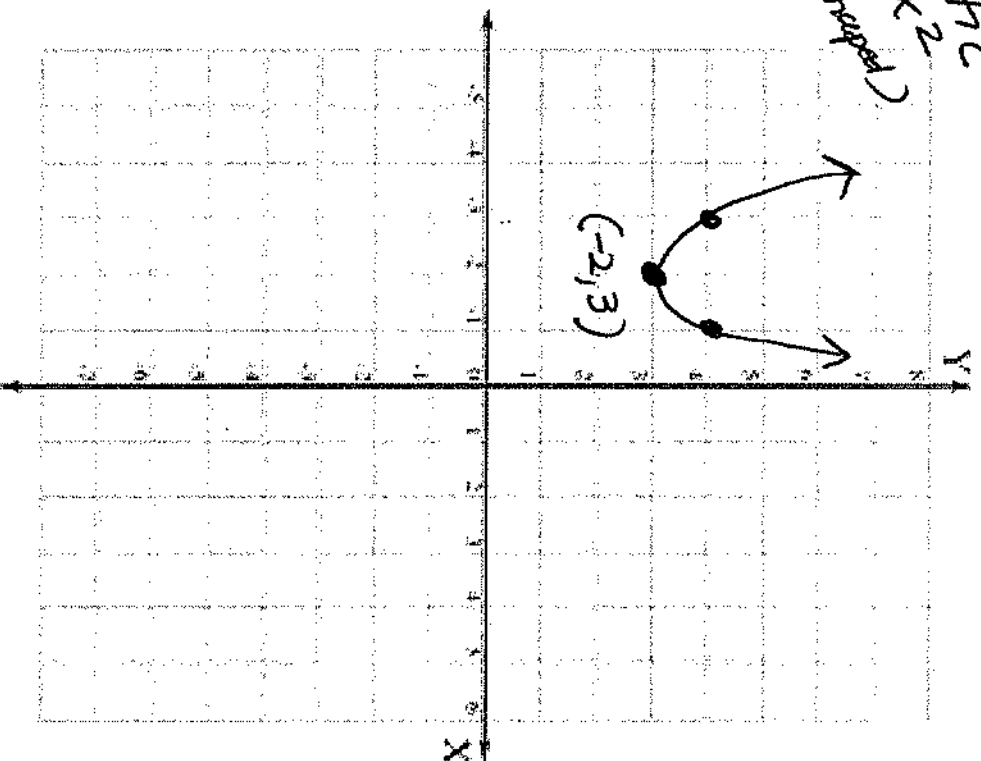
a) $f(x) = (x + 2)^2 + 3$

parabola
Quadratic
 $y = x^2$
(U-shaped)

$x + 2 = 0$
 $x = -2$

$h = -2$

- Up 3
- Left 2



*

x	y
-3	4
-2	3
-1	4

INTERVAL

Domain:	$(-\infty, \infty)$	Range:	$[3, \infty)$
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Sketching SQUARE ROOT Functions

$$f(x) = \sqrt{x}$$

$$f(x) = a\sqrt{b(x - h)} + k$$

Process

1. Identify the key point: (h, k)
2. Make a table of values with two other points, to the right OR left of the key point.

ex: Describe the transformations from the parent function then sketch the square root function. State the D/R in any notation.

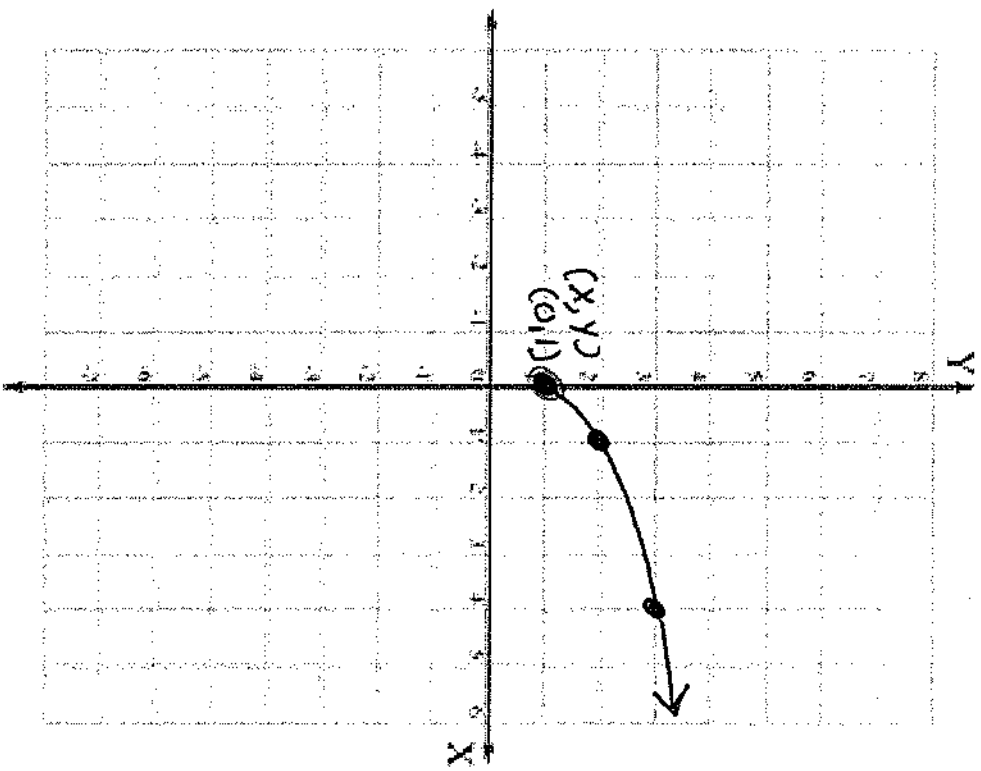
a) $f(x) = \sqrt{x} + 1$

square root

$x=0$

↑
• up 1

X	Y
0	1
1	2
4	3
9	4



INTERVAL

Domain: $[0, \infty)$	Range: $[1, \infty)$
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ex: Describe the transformations from the parent function then sketch the square root function. State the D/R in any notation.

b) $f(x) = 3 - \sqrt{x-2}$

square root

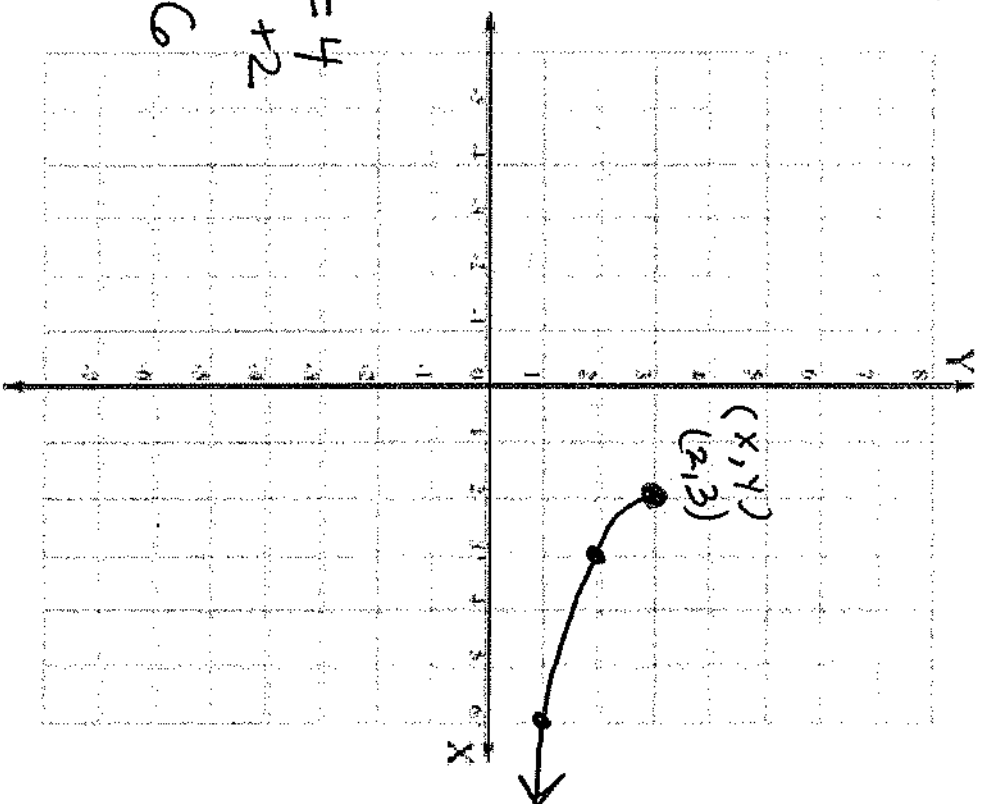
$f(x) = -\sqrt{x-2} + 3$
 \nearrow $h=2$ \nwarrow

$x-2=0$ • X-axis reflection

$x=2$ • right +2 Perfect squares

• Up 3 $x-2=1$ $x-2=4$
 $+2$ $+2$ $+2$ $+2$

$x=3$ $x=6$



X	Y
2	3
3	2
6	1

Domain:

$\{x | x \geq 2\}$

Range:

$\{y | y \leq 3\}$

SET: