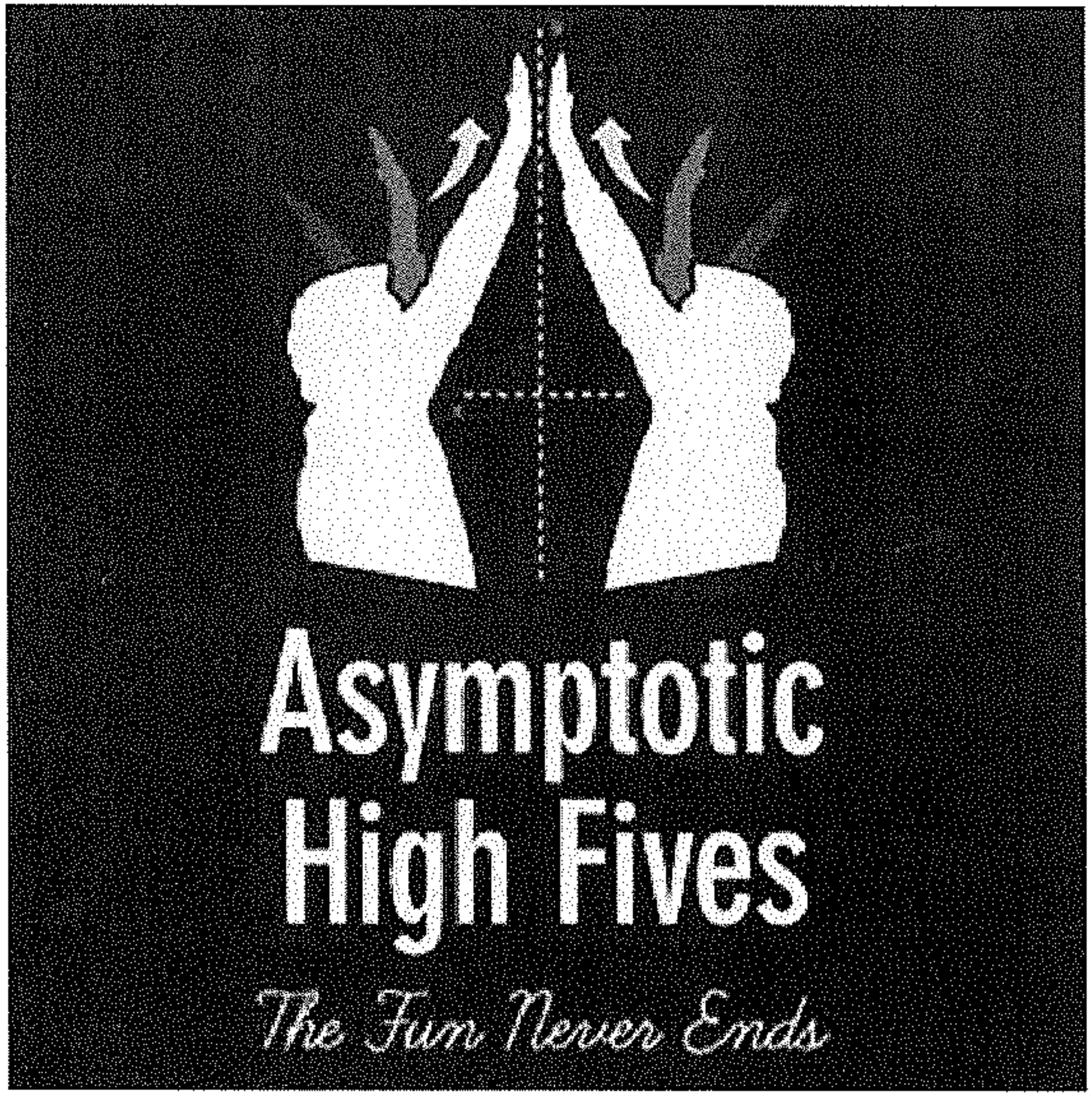
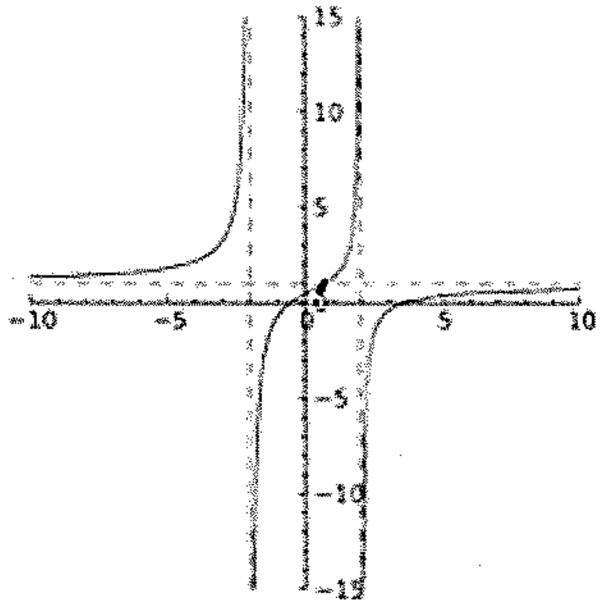
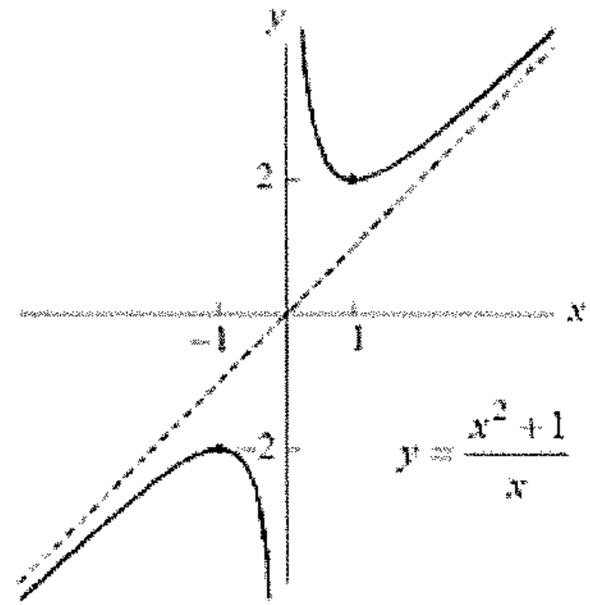
Graphs of Rational Functions

Notes



Graphs of Rational Functions





When Sketching Rational Functions You Must Find:

- x-intercept(s)
- y-intercept
- asymptotes HA ¢ VA
- holes (x,y) o

x-intercepts (plug in 0 for y)

real #'s only

Finding x-intercepts:

Simplify the function; set the numerator equal to zero.

ex: Find the x-intercepts, if any.

a) $y = \dfrac{7x}{x+5}$

$\dfrac{0}{1} = \dfrac{7x}{x+5}$

$\dfrac{\cancel{7}x}{\cancel{7}} = \dfrac{0}{7}$

$x = 0$

x-int:

$\boxed{(0,0)}$

1. $y = 0$
2. * Simplify
3. Cross multiply
4. Solve
5. Coordinates (x, y)

ex: Find the x-intercepts, if any.

b) $g(x) = \dfrac{x^2 + 4x}{x^2 - 16}$

factor to simplify!

hole

$$0 = \frac{x\cancel{(x+4)}}{\cancel{(x+4)}(x-4)}$$

$$\frac{0}{1} = \frac{x}{x-4}$$

$$x = 0$$

x-int:

$\boxed{(0,0)}$

ex: Find the x-intercepts, if any.

c) $f(x) = \dfrac{x^2 - x - 12}{x^2 - 2x - 8}$

hole

$$0 = \frac{\cancel{(x-4)}(x+3)}{\cancel{(x-4)}(x+2)}$$

$$\frac{0}{1} = \frac{x+3}{x+2}$$

$$x+3 = 0$$

$$x = -3$$

x-int

$\boxed{(-3,0)}$

y-intercept

> *Finding the y-intercept:*
> *Plug in $x = 0$ and solve for y.*

ex: Find the y-intercept, if any.

a) $y = \dfrac{7x}{x+5}$

$$y = \frac{7(0)}{0+5}$$

$$y = \frac{0}{5}$$

$$y = 0$$

y-int: $\boxed{(0,0)}$

ex: Find the y-intercept, if any.

b) $g(x) = \dfrac{5}{x}$

$g(0) = \dfrac{5}{0} \leftarrow$ undefined

$\boxed{\text{no y-int.}}$

ex: Find the y-intercept, if any.

c) $g(x) = \dfrac{x^2 + 4x}{x^2 - 16}$

$$g(0) = \frac{0^2 + 4(0)}{0^2 - 16} = \frac{0}{-16} = 0$$

y-int:

$\boxed{(0,0)}$

ex: Find the y-intercept, if any.

d) $f(x) = \dfrac{x^2 - x - 12}{x^2 - 2x - 8}$

$$f(0) = \frac{\cancel{(0)^2} - \cancel{(0)} - 12}{\cancel{(0)^2} - \cancel{2(0)} - 8} = \frac{-12}{-8} = \frac{12}{8} = \frac{3}{2}$$

y-int: $\boxed{(0, 3/2) \text{ or } (0, 1\tfrac{1}{2})}$

Finding Horizontal Asymptotes (HA)

To find the horizontal asymptote, compare the degree of the numerator and denominator. Three cases arise:

Case	Degree		Degree	Asymptote
1	Numerator	<	Denominator	$y=0$
2	Numerator	>	Denominator	no HA
* 3	Numerator	=	Denominator	$y=$ ratio of the leading coefficients

divide the LC's

Rational functions can have at most ONE HA

* one or none (HA)

Remembering Horizontal Asymptotes (HA)

BOBO BOTN *EATSDC

BOBO	BOTN	*EATSDC
Bigger On Bottom,	Bigger On Top	Exponents Are The Same, Divide (leading) Coefficients
$\boxed{y = 0}$	$\boxed{\text{NO HA}}$	$\boxed{y = \text{\_\_\_\_}}$

Horizontal Asymptotes

ex: Find the horizontal asymptote, if any.

a) $y = \dfrac{16x^{1}+1}{4x^{2}-2}$ "BOBO" $\boxed{y=0}$

ratio:

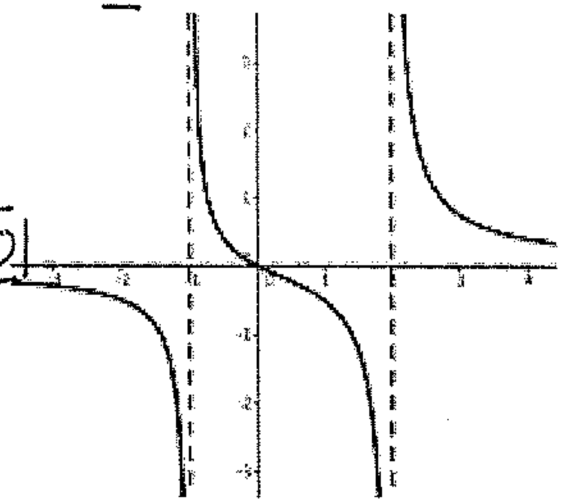
b) $y = \dfrac{16x^{2}+1}{4x^{2}-2}$ "EATS DC" $y = \dfrac{16}{4}$ $\boxed{y=4}$

c) $y = \dfrac{16x^{3}+1}{4x^{2}-2}$ "BOTN" $\boxed{\text{NO HA}}$

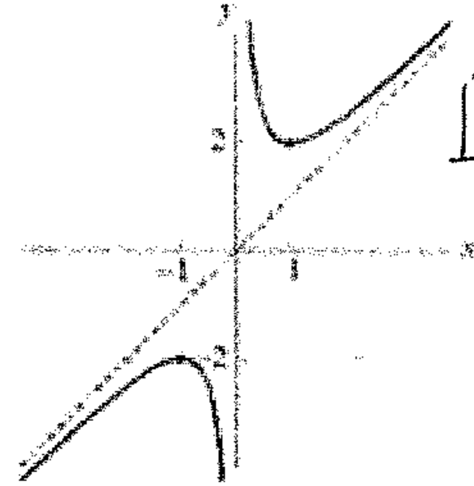
3 examples of graphs: BOBO, BOTN, EATS DC

BOBO

$\boxed{y=0}$

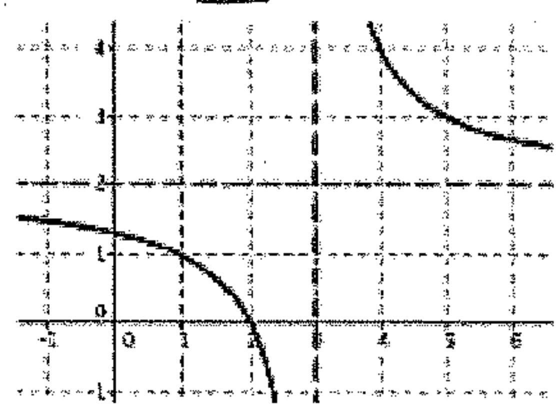


BOTN



$\boxed{\text{NO HA}}$

EATS DC



$\boxed{y=2}$

Finding Vertical Asymptotes (VA) $\updownarrow$

To find vertical asymptotes:

1. Simplify.
2. Set the simplified <u>denominator</u> = 0. Consider real values only.

Rational functions can have more than one VA

* or no VA

Vertical Asymptotes

ex: Find the vertical asymptote(s), if any.

a) $y = \dfrac{x}{3x^2 - 2x - 8}$ factor to simplify

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$\dfrac{3x}{+4}$ $\dfrac{3x}{-6}$ $\div 3$ $\dfrac{1x}{-2}$

$$Y = \frac{x}{(3x+4)(x-2)}$$

nothing cancels (no holes)

now set denominator equal to "0".

$$(3x+4)(x-2) = 0$$

$3x + 4 = 0$ | $x - 2 = 0$

VA: $\boxed{x = \frac{-4}{3}}$ | $\boxed{x = 2}$

ex: Find the vertical asymptote(s), if any.

b) $y = \dfrac{x+1}{x^2+16x+15}$ factor

$$y = \frac{\overset{1}{\cancel{x+1}}}{(x+15)\cancel{(x+1)}}$$

hole

$$y = \frac{1}{x+15}$$

$$x+15 = 0$$

VA: $\boxed{x=-15}$

ex: Find the vertical asymptote(s), if any.

c) $y = \dfrac{x^2}{x^2+9}$

$$x^2+9=0$$

$$\sqrt{x^2} = \sqrt{-9}$$

$$|x| = 3i$$

$$x = \pm 3i \leftarrow \text{imaginary}$$

$$\boxed{\text{no VA}}$$

ex: Find the vertical asymptote(s), if any.

d) $y = \dfrac{6x}{x-7}$

$x - 7 = 0$

VA: $\boxed{x = 7}$

Finding Holes (x, y) o open on the graph

To find holes:

1. Factor completely.
2. If the numerator and denominator share a common factor a hole exists.
3. The hole exists at the zero of the common factor. $x =$

* 4. To find the y-value, plug in x into the SIMPLIFIED version, of the equation.

Rational functions can have more than one hole

or no holes

ex: Find all holes, if any.

a) $y = \dfrac{x^2 - 4}{x^2 - x - 2}$

$$y = \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}(x+1)}$$

hole

$$y = \frac{x+2}{x+1}$$

Hole:

$x - 2 = 0$

$x = 2$

- now plug into the simplied equation

$$y = \frac{2+2}{2+1} = \frac{4}{3}$$

Hole: $\boxed{(2, 4/3)}$

ex: Find all holes, if any.

b) $y = \dfrac{x+5}{5x^2 - 3x - 2}$ factor

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$\dfrac{5x}{+2}$ $\dfrac{5x}{-5}$

$\div 5$

$$y = \frac{x+5}{(5x+2)(x-1)}$$

nothing cancels

$\boxed{\text{no holes}}$

ex: Find all holes, if any.

c) $y = \dfrac{5x^2 - 9x - 18}{x^2 - 3x}$ factor

$$y = \frac{(5x+6)\cancel{(x-3)}}{x\cancel{(x-3)}}$$ hole

$$y = \frac{5x+6}{x}$$

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1	90
2	45
3	30
5	18
$\frac{5x}{+6}$	$\frac{5x}{-15}$
	$\div 5$

Hole:

$x - 3 = 0$

$x = 3$

$$y = \frac{5(3)+6}{3} = \frac{21}{3} = 7$$

Hole: $\boxed{(3,7)}$

ex: Find all holes, if any.

d) $y = \dfrac{x^3 + x^2 + 3x + 3}{x^2 - 1}$ factor

grouping:

$x^2\underline{(x+1)} + 3\underline{(x+1)}$

$(x+1)(x^2+3)$

hole

$y = \dfrac{\cancel{(x+1)}(x^2+3)}{\cancel{(x+1)}(x-1)}$

$y = \dfrac{x^2+3}{x-1}$

Hole:

$x+1=0$

$x=-1$

$y = \dfrac{(-1)^2+3}{(-1)-1}$

$y = \dfrac{1+3}{-1-1} = \dfrac{4}{-2} = -2$

Hole: $\boxed{(-1,-2)}$