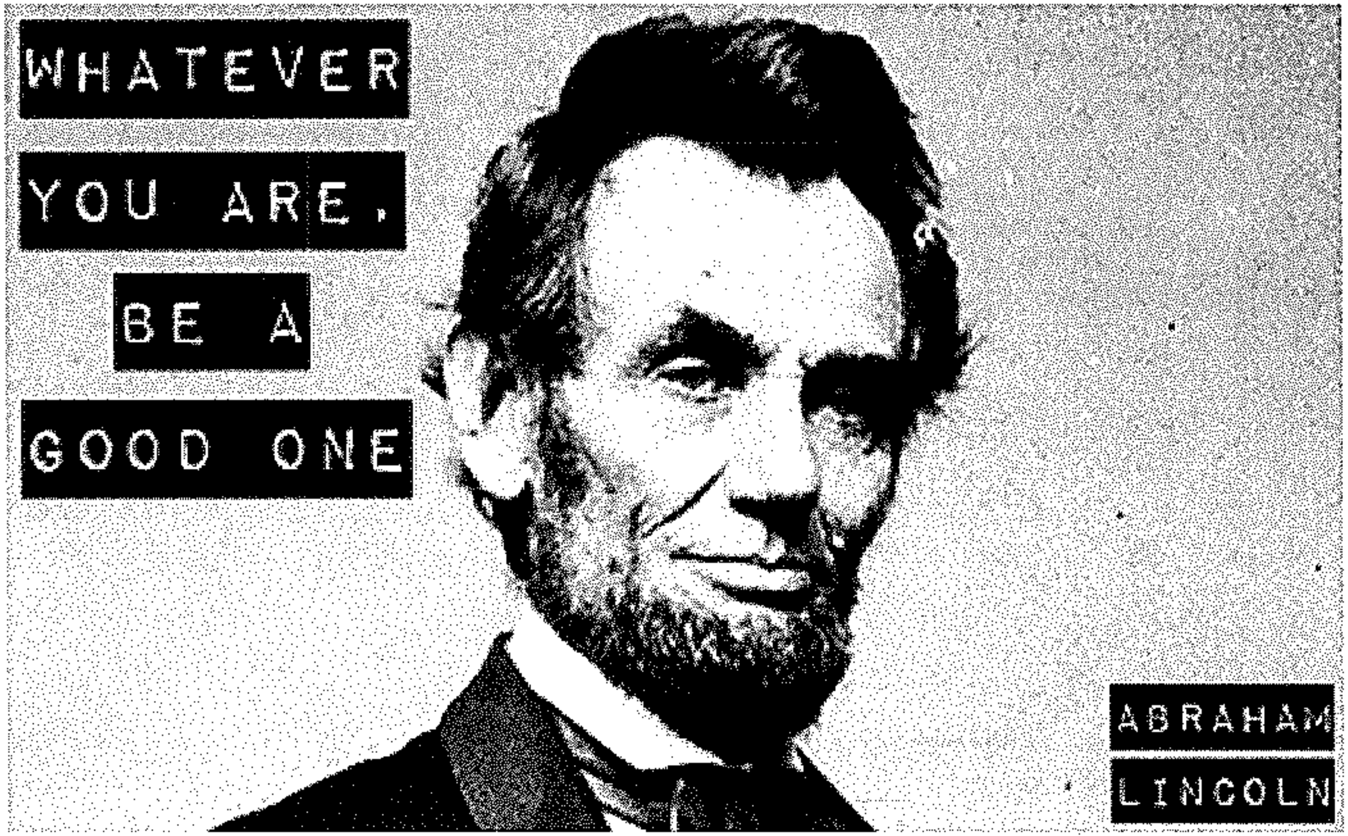
A2: Graphs of Exponential Functions

Notes



Exponential Functions

growth or decay factor

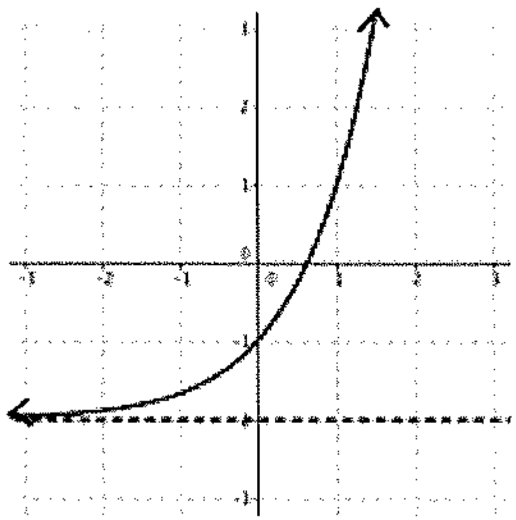
$$f(x) = ab^x$$

$$a \neq 0, \quad b > 0, \quad b \neq 1$$

b is called the growth or decay factor

Graphs of Exponential Functions

$$f(x) = a\underline{\underline{b}}^x$$

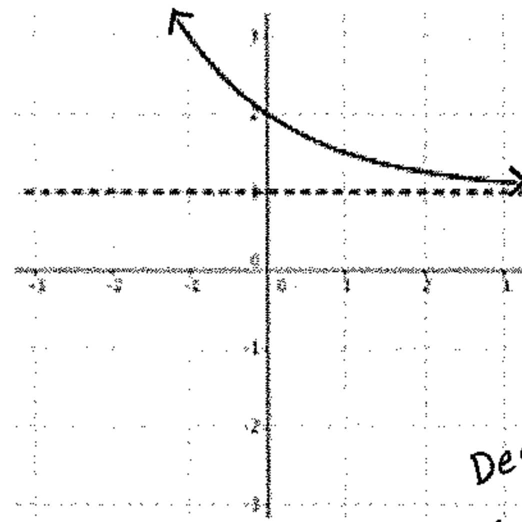


Increases from L to R

<u>Exponential Growth</u>

$\underline{\underline{b}} > 1$

*the RIGHT side of the graph moves AWAY from the asymptote



Decreases from L to R

<u>Exponential Decay</u>

$0 < \underline{\underline{b}} < 1$

*the RIGHT side of the graph moves TOWARDS the asymptote

ex: Sketch. Then state the domain and range and classify as growth or decay.

SET: $\{x \mid x \in \mathbb{R}\}$	$\{y \mid y > 4\}$
INT: $(-\infty, \infty)$	$(4, \infty)$

a) $y = 2 \cdot 3^{x-1} + 4$

set = 0 to begin the table

$3 > 1$

$\boxed{\text{Growth}}$ factor

HA = 4

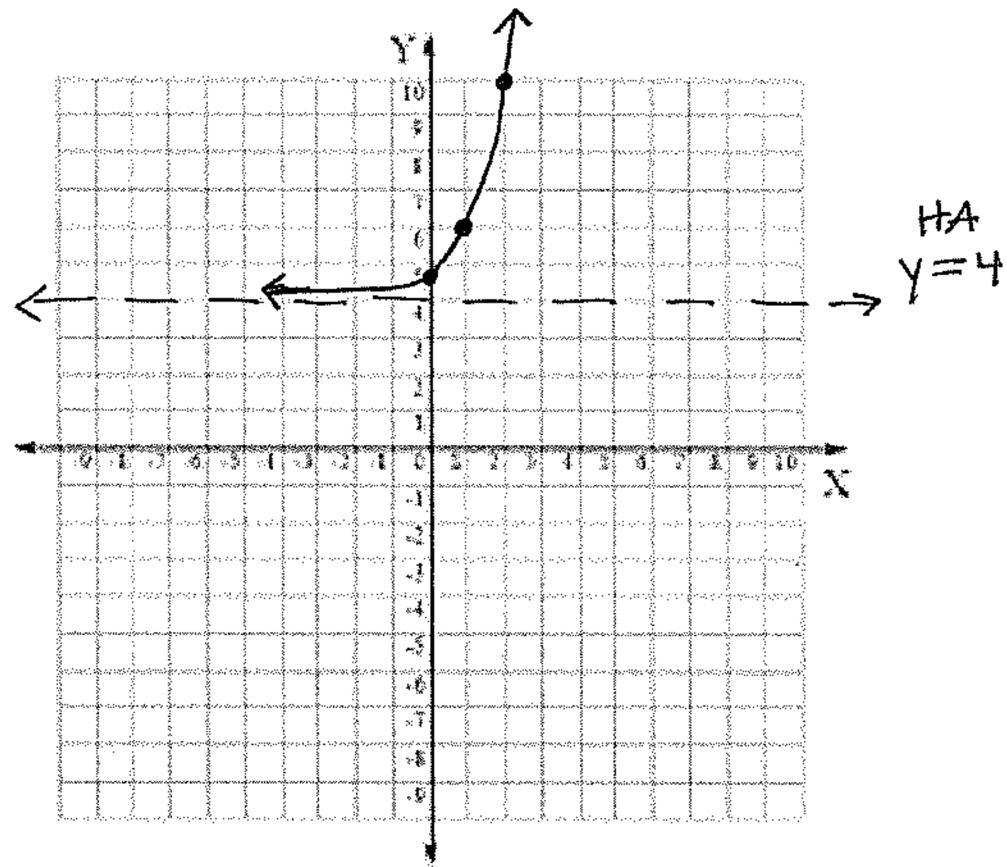
$y = 4$

x	y	
0	$4\frac{2}{3}$	y-int
1	6	start
2	10	

$x - 1 = 0$

$x = 1$

$y = 2 \cdot 3^{(1-1)} + 4$
$= 2 \cdot 3^0 + 4$
$= 2 \cdot 1 + 4$
$= 2 + 4$
$= 6$



$x = 2$

$y = 2 \cdot 3^{(2-1)} + 4$
$= 2 \cdot 3^1 + 4$
$= 6 + 4$
$= 10$

$x = 0$

$y = 2 \cdot 3^{(0-1)} + 4$
$= 2 \cdot 3^{-1} + 4$
$= 2 \cdot \frac{1}{3} + 4$
$= \frac{2}{3} + 4 \Rightarrow 4\frac{2}{3}$

ex: Sketch. Then state the domain and range and classify as growth or decay.

	Domain	Range
SET:	$\{x \mid x \in \mathbb{R}\}$	$\{y \mid y > -1\}$
INT:	$(-\infty, \infty)$	$(-1, \infty)$

b) $y = 3\left(\frac{1}{2}\right)^{x+2} \underbrace{-1}$

set = 0 for the table

HA = −1
$y = -1$

$0 < \frac{1}{2} < 1$

$\boxed{\text{Decay}}$ factor

X	Y
−3	5
−2	2
−1	½

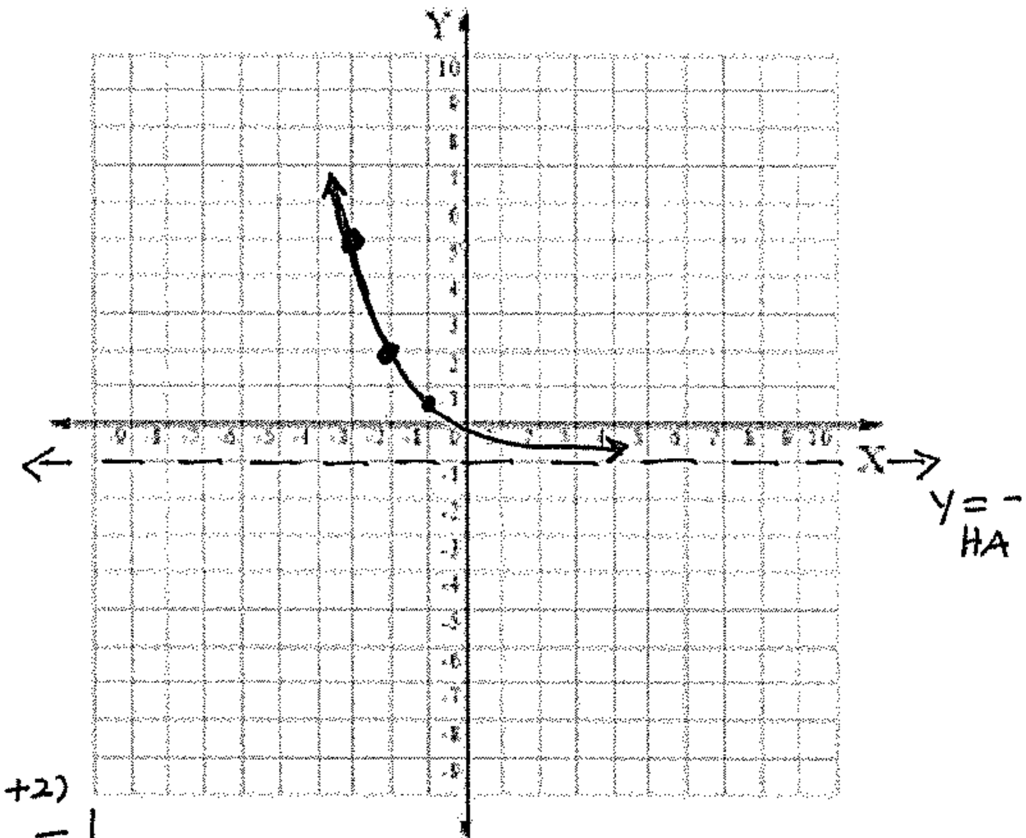
$x + 2 = 0$

$x = -2$

$y = 3\left(\frac{1}{2}\right)^{(-2+2)} - 1$
$= 3\left(\frac{1}{2}\right)^{0} - 1$
$= 3(1) - 1$
$= 3 - 1$
$= 2$

$x = -1$

$y = 3\left(\frac{1}{2}\right)^{(-1+2)} - 1$
$= 3\left(\frac{1}{2}\right)^{1} - 1$
$= \frac{3}{2} - 1$
$= \frac{3}{2} - \frac{2}{2} = \frac{1}{2}$



$x = -3$

$y = 3\left(\frac{1}{2}\right)^{(-3+2)} - 1$
$= 3\left(\frac{1}{2}\right)^{(-1)} - 1$
$= 3\left(\frac{2}{1}\right)^{1} - 1$
$= 3(2) - 1 \Rightarrow 6 - 1 \Rightarrow 5$

ex: Sketch. Then state the domain and range and classify as growth or decay.

	Domain	Range
SET	$\{x \mid x \in \mathbb{R}\}$	$\{y \mid y < 0\}$
INT	$(-\infty, \infty)$	$(-\infty, 0)$

SET = 0

c) $y = -\left(\frac{2}{3}\right)^{x} + 0$

HA = 0
y = 0

$0 < \frac{2}{3} < 1$

$\boxed{\text{Decay}}$ factor

X	Y
-1	$-\frac{3}{2} = -1\frac{1}{2}$
0	-1
1	$-\frac{2}{3}$

$x = 0$

$y = -\left(\frac{2}{3}\right)^0$

$= -1 \cdot 1$

$= -1$

$x = 1$

$y = -\left(\frac{2}{3}\right)^1$

$= -1 \cdot \frac{2}{3}$

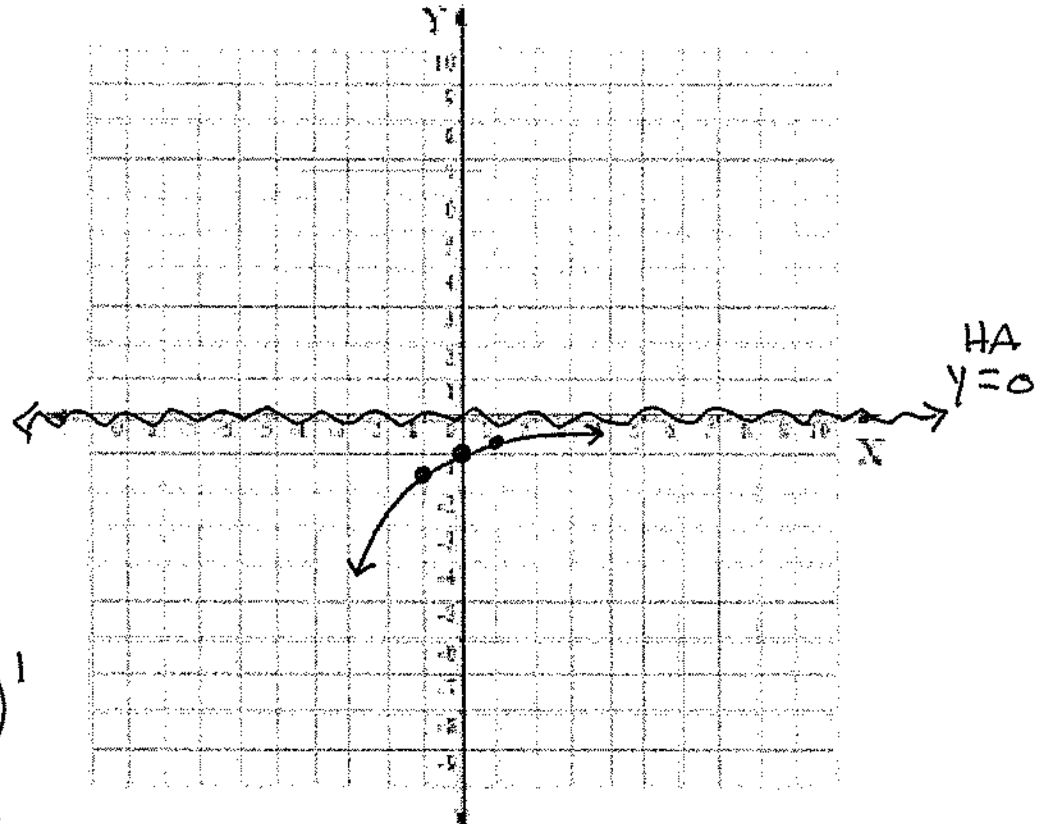
$= -\frac{2}{3}$

$x = -1$

$y = -\left(\frac{2}{3}\right)^{-1}$

$= -1 \cdot \left(\frac{3}{2}\right)^1$

$= -\frac{3}{2} = -1\frac{1}{2}$



ex: Sketch. Then state the domain and range and classify as growth or decay.

$\{x \mid x \in \mathbb{R}\}$	$\{y \mid y > 0\}$
$(-\infty, \infty)$	$(0, \infty)$

d) $y = 2 \cdot 5^{4-x} + 0$

set = 0

$5 > 1$

$\boxed{\text{Growth}}$ factor

HA = 0

y = 0

x	y
3	10
* 4	2
5	$\frac{2}{5}$

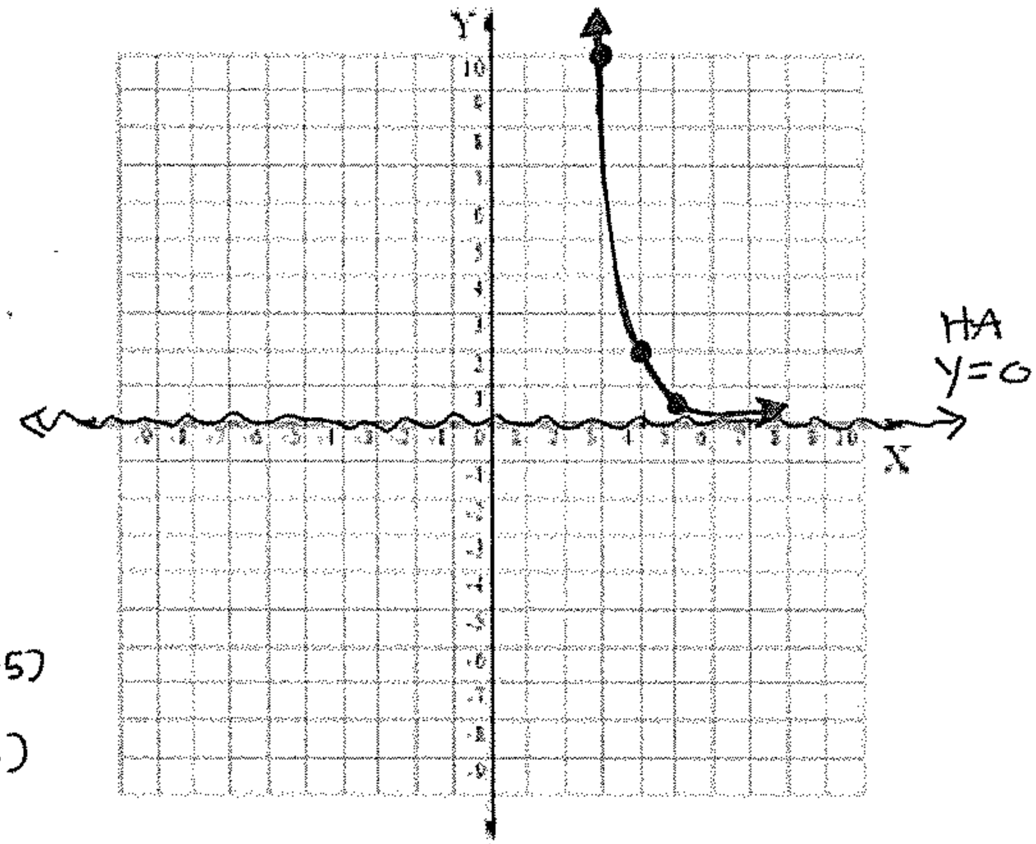
$4 - x = 0$

$4 = x$

$$y = 2 \cdot 5^{(4-4)}$$
$$= 2 \cdot 5^0$$
$$= 2 \cdot 1$$
$$= 2$$

$x = 5$

$$y = 2 \cdot 5^{(4-5)}$$
$$= 2 \cdot 5^{(-1)}$$
$$= 2 \cdot \frac{1}{5}$$
$$= \frac{2}{5}$$



$x = 3$

$$y = 2 \cdot 5^{(4-3)}$$
$$= 2 \cdot 5^1$$
$$= 10$$

ex: Sketch. Then state the domain and range and classify as growth or decay.

Domain	Range
$\{x \mid x \in \mathbb{R}\}$ $(-\infty, \infty)$	$\{y \mid y > -1\}$ $(-1, \infty)$

e) $y = e^{x+3} - 1$

set = 0

$e > 1$

$\boxed{\text{Growth}}$ factor

HA = −1
$y = -1$

x	y
−4	−0.632
−3	0
−2	1.718

$x + 3 = 0$

$x = -3$

$y = e^{(-3+3)} - 1$
$= e^0 - 1$
$= 1 - 1$
$= 0$

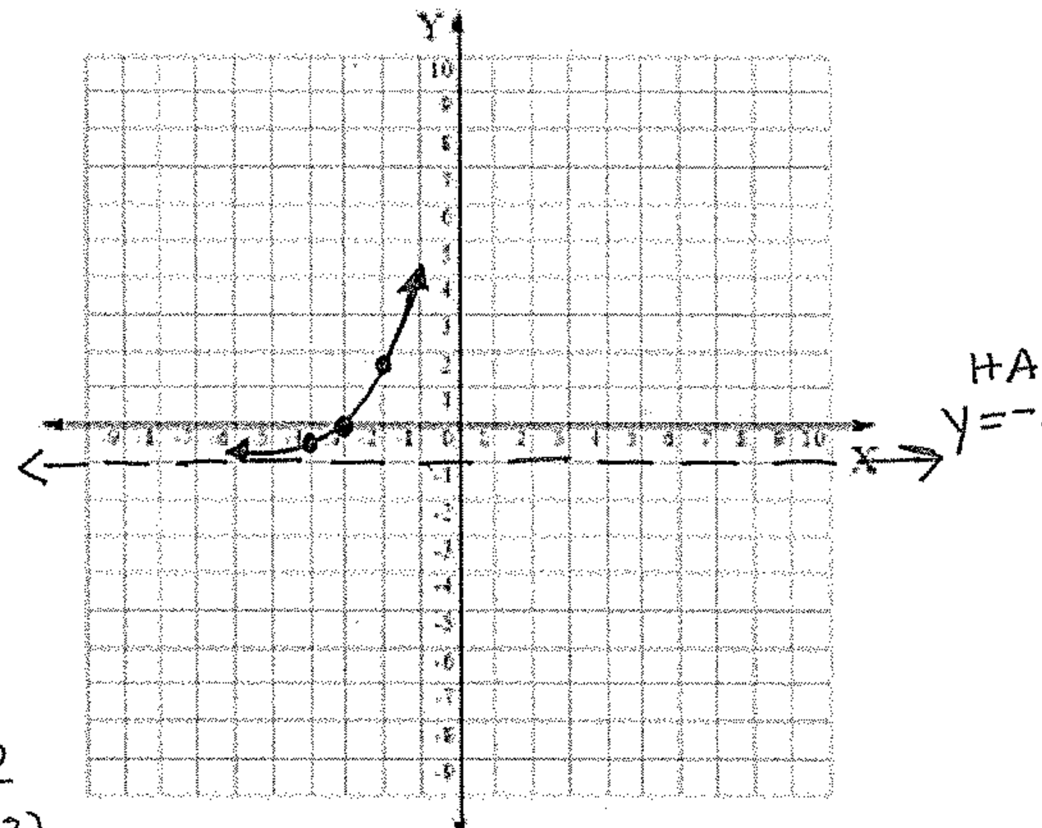
$x = -2$

$y = e^{(-2+3)} - 1$
$= e^1 - 1$
$= e - 1 \approx 1.718$

use a calculator

$x = -4$

$y = e^{(-4+3)} - 1$
$= e^{-1} - 1$
$= \frac{1}{e} - 1 \approx -0.632$



ex: WITHOUT graphing determine if the function represents growth or decay, then state the growth or decay factor.

a) $y = \frac{1}{2} \cdot 3^{x-4} + 5$

$3 > 1$

$\boxed{\text{Growth}}$

Growth factor $= \boxed{3}$

ex: WITHOUT graphing determine if the function represents growth or decay, then state the growth or decay factor.

b) $y = -\left(\frac{4}{5}\right)^{x+1}$

$0 < \frac{4}{5} < 1$

$\boxed{\text{Decay}}$

Decay factor $= \boxed{\frac{4}{5}}$