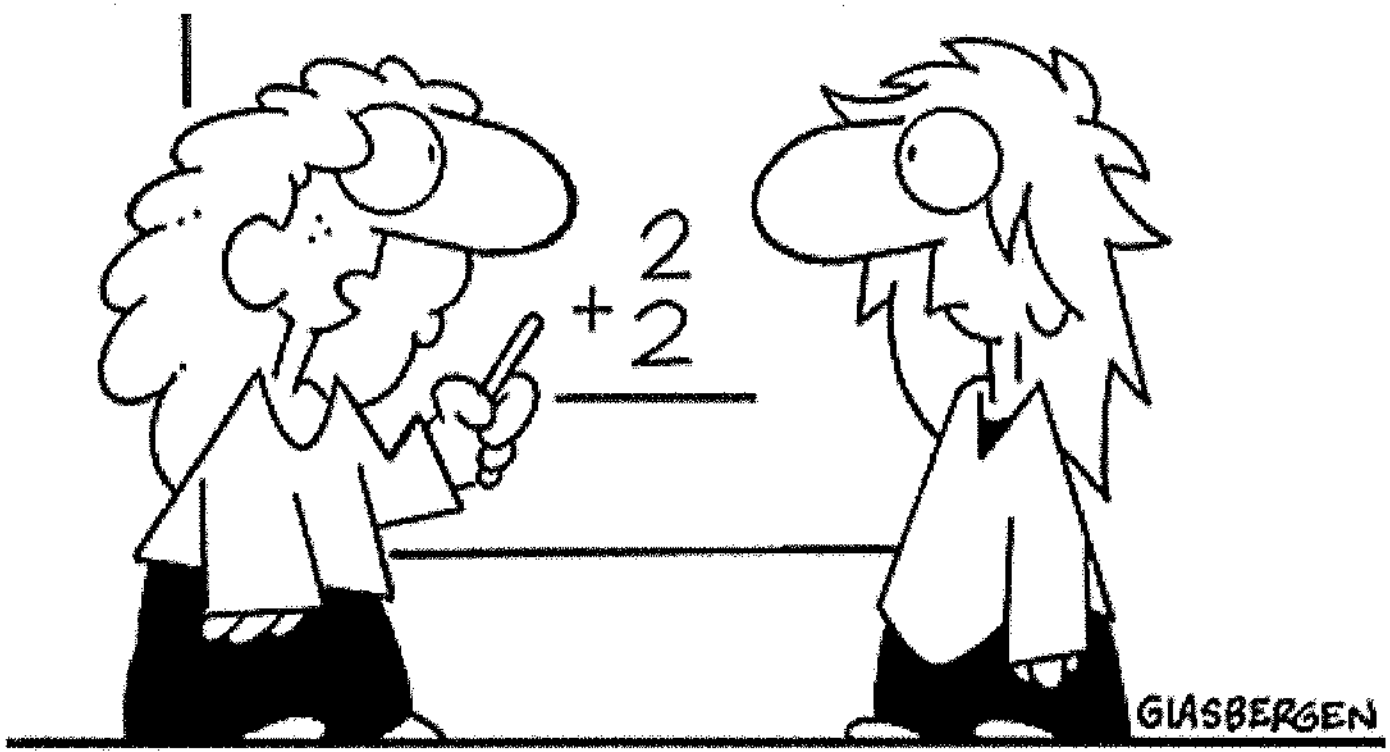
# A2 Graphs of Rational Functions - cont.

# Notes



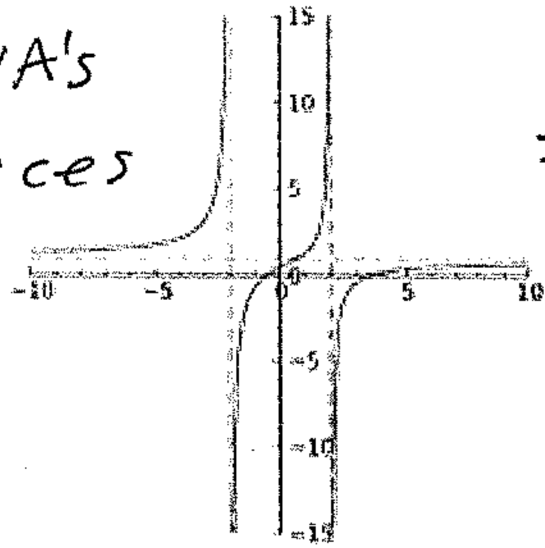

"First they build up your confidence with simple addition and subtraction, then they slam you with algebra and calculus. It's quite a clever scheme."

Sketching Requirements:

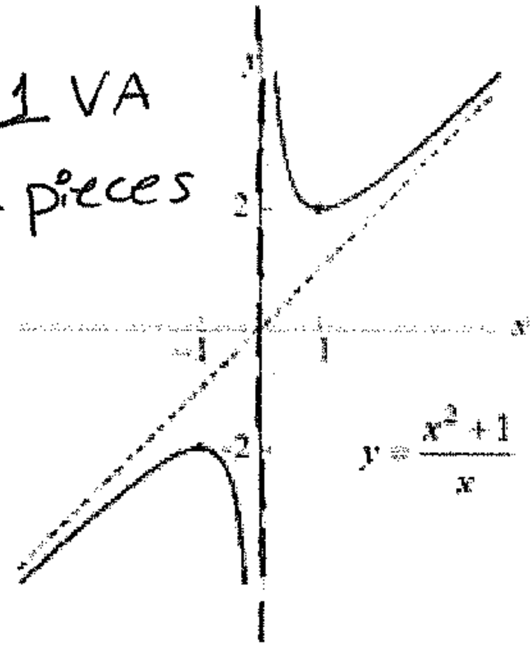
- x-intercepts
- y-intercept
- asymptotes HA & VA
- holes $(x, y)$ o ← open on graph
- plot at least one point per "piece"

# Number of VA and the Number of "Pieces"

2 VA's
= 3 pieces




1 VA
= 2 pieces




| Number of Vertical Asymptotes | Number of "Pieces" |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 3 |
| n | n+1 |

ex: Sketch and state the domain, range and end behavior.

a) $y = \dfrac{2x^{①} - 4}{x^{①} - 3} = \dfrac{2(x-2)}{x-3}$ ← VA: $x - 3 = 0$ $\boxed{x=3}$

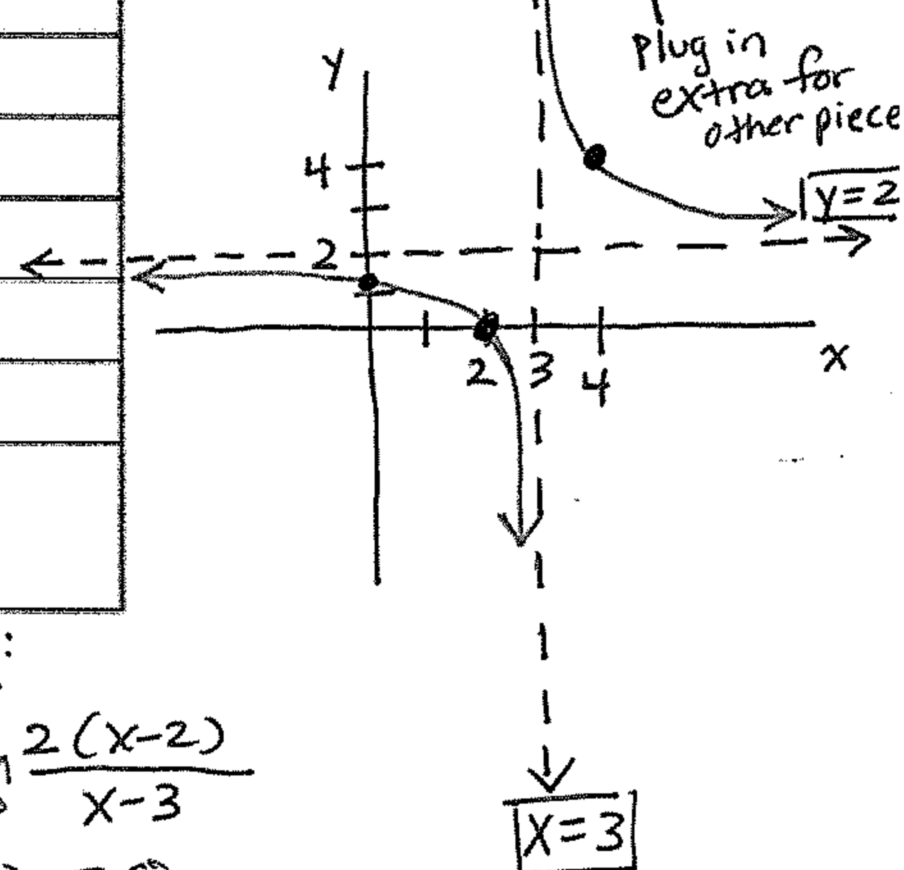
nothing cancels = no holes

1 VA = 2 pieces

| | |
|---|---|
| ④ x-int | $(2, 0)$ |
| ① y-int | $(0, 4/3)$ or $(0, 1\tfrac{1}{3})$ |
| ③ VA ● | $\boxed{x = 3}$ |
| ① HA ■? | EATS DC $y = \frac{2}{1}$ $\boxed{y = 2}$ |
| ② Holes ● ■ | none |
| Domain ● | $\{x \mid x \neq 3\}$ |
| * Range ■ | $\{y \mid y \neq 2\}$ |
| End Behavior (HA) | $x \to -\infty,\ y \to 2$<br>$x \to \infty,\ y \to 2$ |

| x | y |
|---|---|
| 4 | $\frac{4}{1} = 4$ |

Plug in extra for other piece




y-int: $\dfrac{2(0)-4}{0-3} = \dfrac{-4}{-3} = \dfrac{4}{3}$ $\boxed{(0, \tfrac{4}{3})}$

X-int: $\dfrac{0}{1} = \dfrac{2(x-2)}{x-3}$

$2(x-2) = 0$

$x = 2$ $\boxed{(2, 0)}$

ex: Sketch and state the domain, range and end behavior.

b) $f(x) = \dfrac{x^2 + 3x + 2}{x^2 - 4x - 5} = \dfrac{(x+2)(x+1)}{(x-5)(x+1)}$ hole → Hole: $x+1=0$, $x=-1$

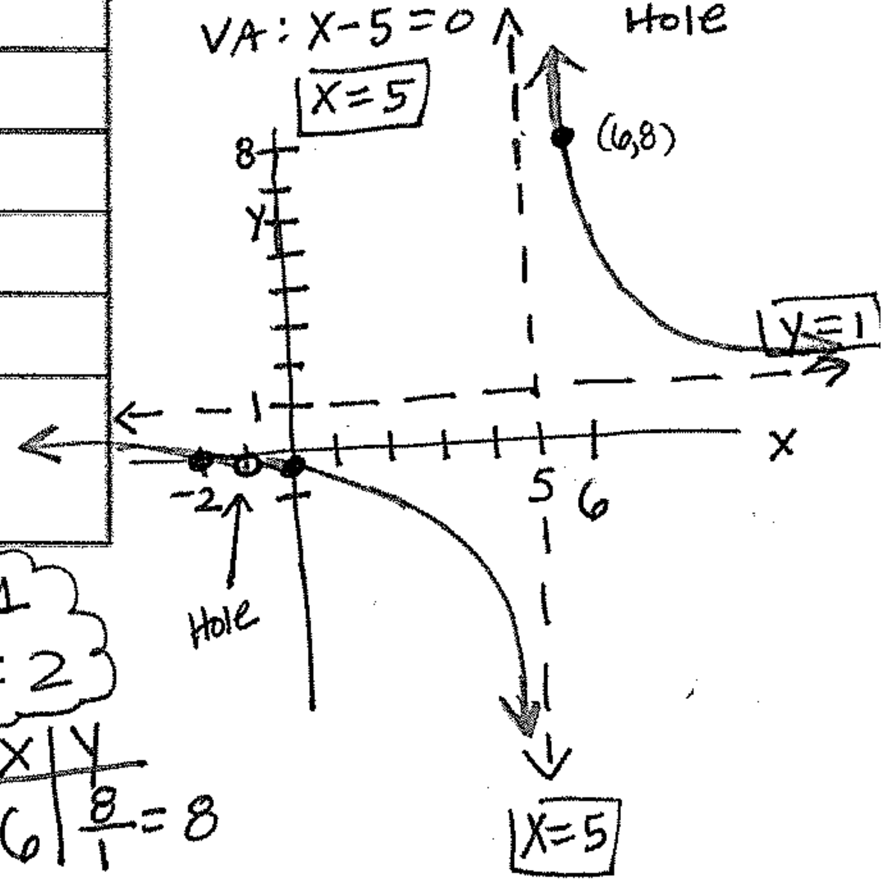
$f(-1) = \dfrac{-1+2}{-1-5} = \dfrac{1}{-6}$

$(-1, -1/6)$ Hole

$f(x) = \dfrac{x+2}{x-5}$

VA: $x-5=0$, $x=5$

| | |
|---|---|
| (4) x-int | $(-2, 0)$ |
| (1) y-int | $(0, -2/5)$ |
| (3) VA • | $x=5$ |
| (1) HA ■ ? | EATS DC $y=\frac{1}{1}$  $y=1$ |
| (2) Holes • ■ | $(-1, -1/6)$ o ← open |
| Domain • | $\{x \mid x \neq -1, 5\}$ |
| * Range ■ | $\{y \mid y \neq -1/6, 1\}$ |
| End Behavior (HA) | $x \to -\infty$, $y \to 1$<br>$x \to \infty$, $y \to 1$ |




VA: 1
Pieces: 2

| x | y |
|---|---|
| 6 | $\frac{8}{1} = 8$ |

x-int:

$\frac{0}{1} = \frac{x+2}{x-5}$  $(-2, 0)$

$x+2=0$  $x=-2$

ex: Sketch and state the domain, range and end behavior..

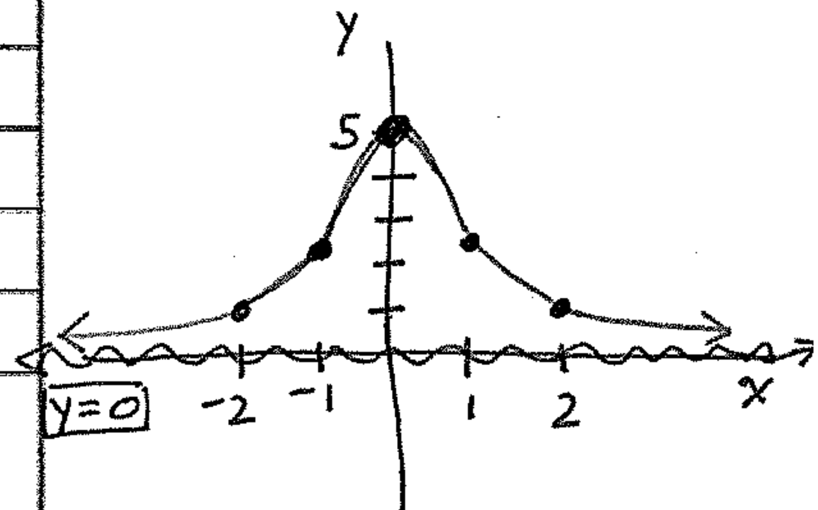
c) $f(x) = \frac{5}{x^{2}+1}$ ← VA: $x^2+1=0$

$\sqrt{x^2} = \sqrt{-1}$ ← imaginary

no VA

0 VA = 1 piece

| | |
|---|---|
| x-int | none |
| y-int | (0,5) |
| VA • | none |
| HA | BOBO $y=0$ |
| Holes • | none |
| Domain • | $\{x \mid x \in \mathbb{R}\}$ |
| * Range | $\{y \mid 0 < y \leq 5\}$ |
| End Behavior (HA) | $x \to -\infty,\ y \to 0$<br>$x \to \infty,\ y \to 0$ |




x-int:

$0 = \frac{5}{x^2+1}$

$5 \neq 0$ no x-int

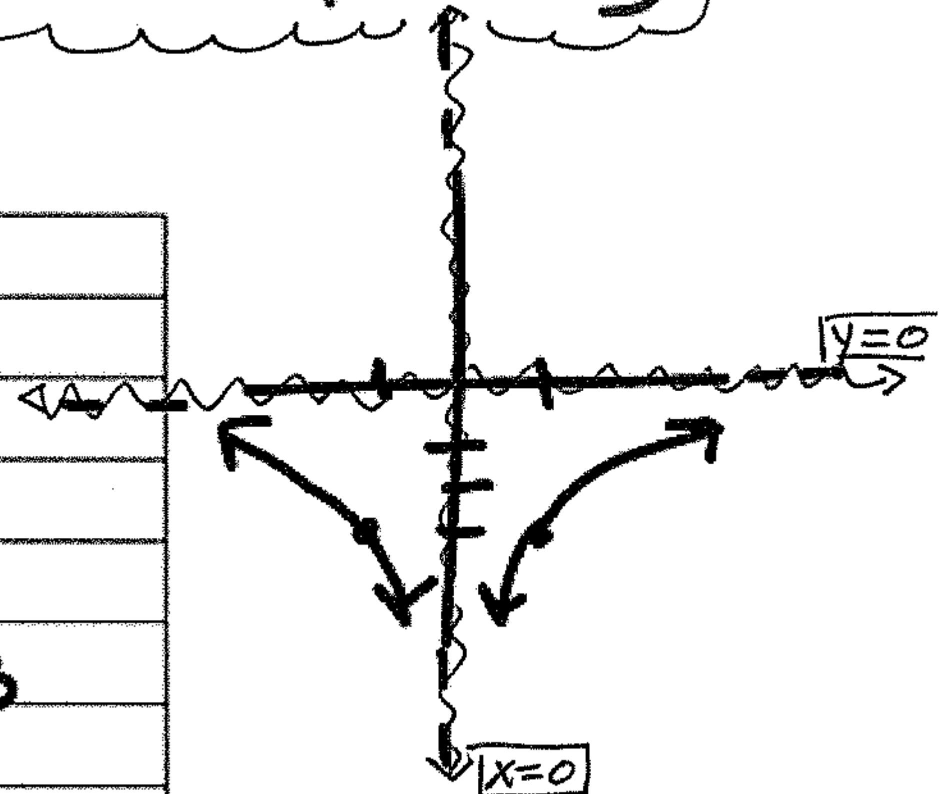
| x | y |
|---|---|
| 1 | $\frac{5}{2} = 2.5$ |
| -1 | $\frac{5}{2} = 2.5$ |

ex: Sketch and state the domain, range and end behavior..

d) $y = \frac{-3}{x^2}$

1VA = 2 pieces

| | |
|---|---|
| x-int | none |
| y-int | none |
| VA | $x = 0$ |
| HA | $y = 0$ |
| Holes | none |
| Domain | $\{x \mid x \neq 0\}$ |
| * Range * | $\{y \mid y < 0\}$ |
| End Behavior (HA) | $x \to -\infty$ $y \to 0$<br>$x \to \infty$ $y \to 0$ |




| x | y |
|---|---|
| -1 | -3 |
| 1 | -3 |

ex: Sketch and state the domain, range and end behavior..

e) $y = \frac{3x^{①} - 3}{x^{②} - 4x} = \frac{3(x-1)}{x(x-4)}$ ← VA: $x(x-4) = 0$

$x=0$ | $x=4$

2 VA = 3 pieces

| | |
|---|---|
| x-int | (1, 0) mult: 1 (cross) |
| y-int | none |
| VA • | $x=0$ $x=4$ |
| HA | BOBO $y=0$ |
| Holes • | none |
| Domain • | $\{x \mid x \neq 0, 4\}$ |
| * Range * | $\{y \mid y \in \mathbb{R}\}$ crossed HA |
| End Behavior HA | $x \to -\infty, y \to 0$<br>$x \to \infty, y \to 0$ |

y-int:

$y = \frac{3(0) - 3}{0^2 - 4(0)} = \frac{-3}{0}$ und.

no y-int

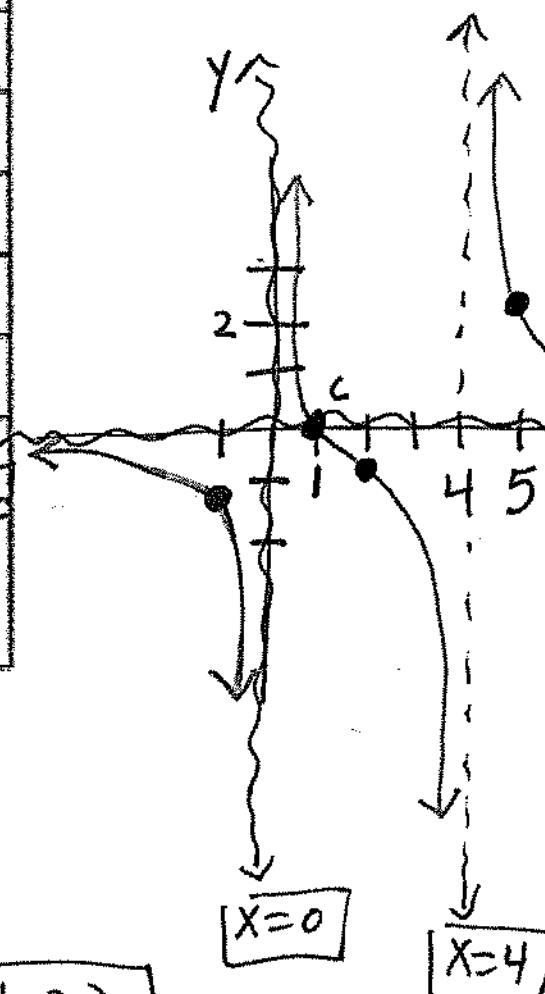
x-int:

$\frac{0}{1} = \frac{3(x-1)}{x(x-4)}$

$3(x-1) = 0$

$x = 1$

(1, 0)




| x | y |
|---|---|
| 5 | $\frac{12}{5} = 2\frac{2}{5}$ |
| -1 | $\frac{-6}{5} = -1$ |
| 2 | $-\frac{3}{4}$ |

ex: Sketch and state the domain, range and end behavior..

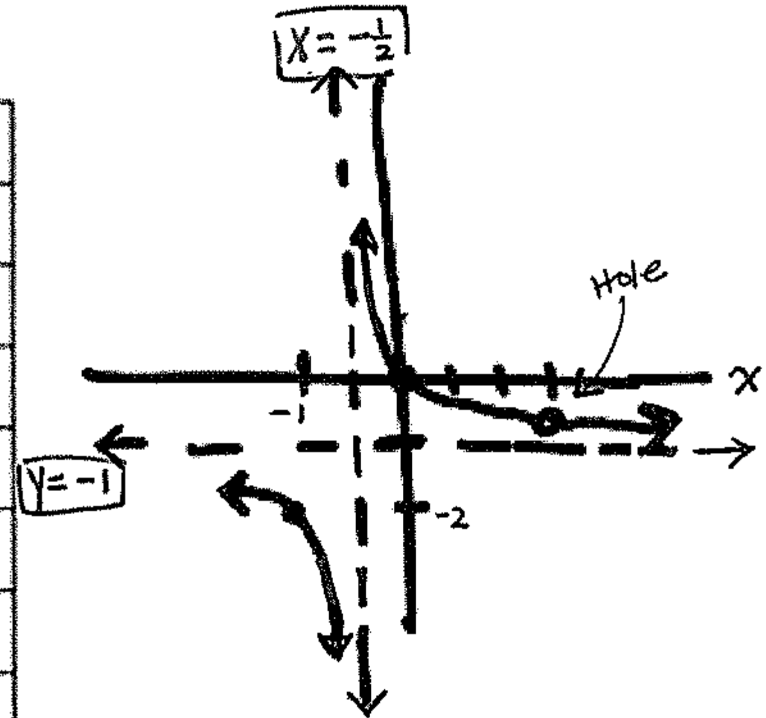
f) $f(x) = \dfrac{-3x^2}{x^2 - 4}$

| | |
|---|---|
| x-int | |
| y-int | |
| VA | |
| HA | |
| Holes | |
| Domain | |
| Range | |
| End Behavior | |

ex: Sketch and state the domain, range and end behavior..

g) $f(x)=\dfrac{-2x^2+6x}{2x^2-5x-3} = \dfrac{-2x(x-3)}{(2x+1)(x-3)} = \dfrac{-2x}{2x+1}$

| | |
|---|---|
| x-int | (0,0) |
| y-int | (0,0) |
| VA • | $x=-1/2$ |
| HA | $y=-1$ |
| Holes • | $(3, -6/7)$ |
| Domain • | $\{x \mid x \neq -1/2, 3\}$ |
| Range | $\{y \mid y \neq -1, -6/7\}$ |
| End Behavior (HA) | $x \to -\infty,\ y \to -1$<br>$x \to \infty,\ y \to -1$ |




1VA = 2 pieces

| x | y |
|---|---|
| -1 | $2/-1 = -2$ |