# A2: Sketching Logarithmic Functions
# Solving Exponential Equations

Notes

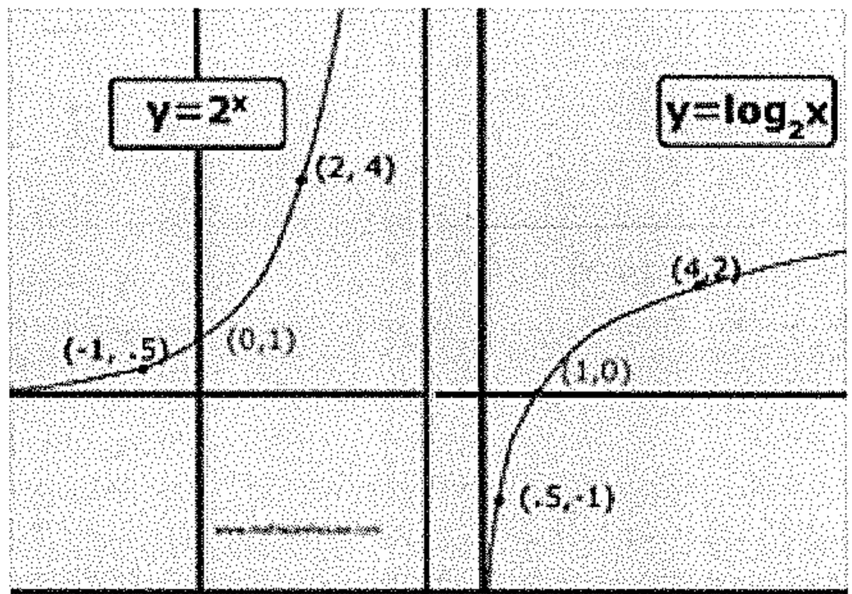
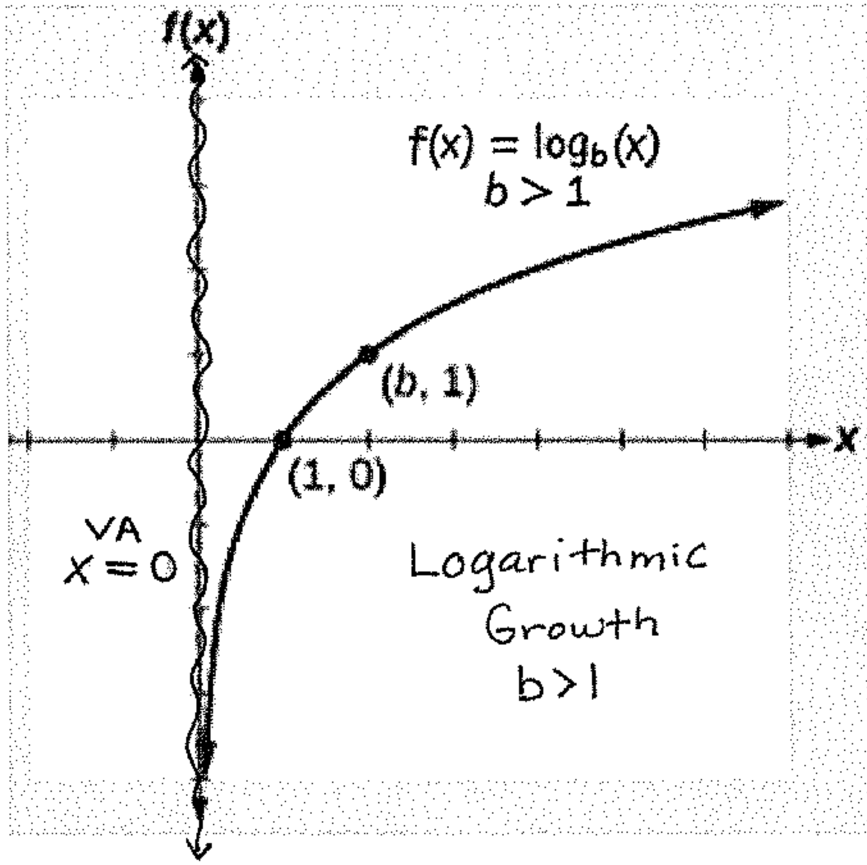


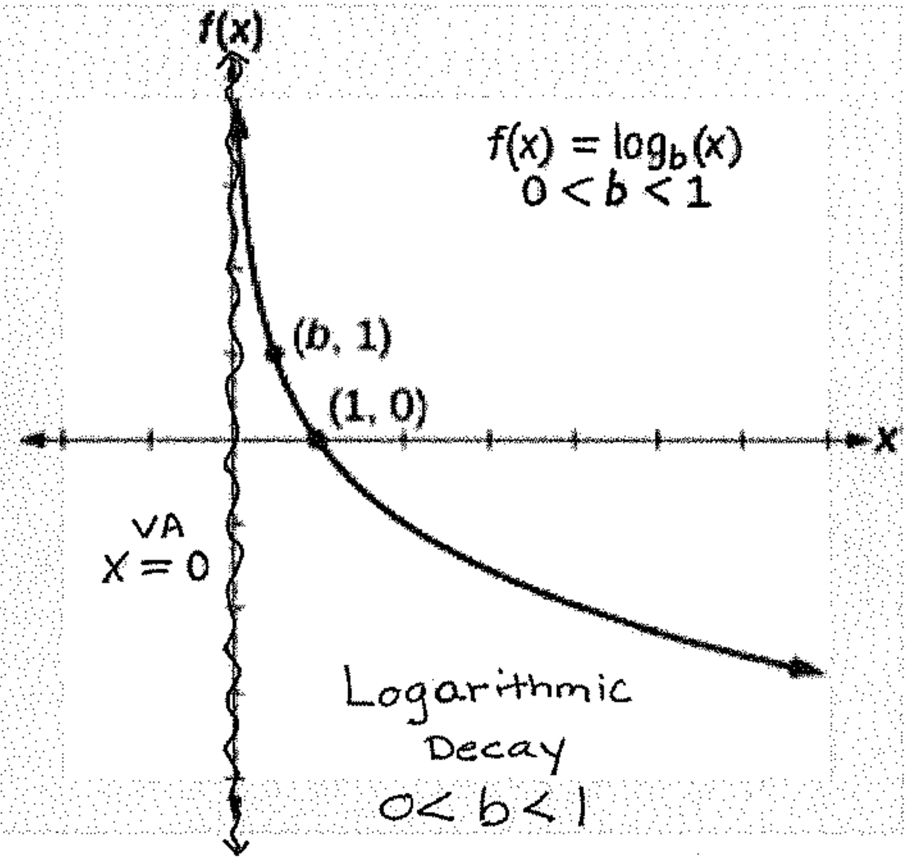

*Table of values*

| x | y |
|---|---|
| (-2 | .25) |
| (-1 | .5) |
| (0 | 1 ) |
| (1 | 2) |
| (2 | 4) |
| (3 | 8) |
| (4 | 16) |
| (5 | 32 ) |

*Table of values*

| x | y |
|---|---|
| (.25 | -2 ) |
| ( .5 | -1) |
| (1 | 0 ) |
| (2 | 1 ) |
| (4 | 2) |
| (8 | 3) |
| (16 | 4) |
| (32 | 5 ) |


f(x)
f(x) = log_b(x)
b > 1
(b, 1)
(1, 0)
x
VA
x = 0
Logarithmic
Growth
b>1


f(x)
f(x) = log_b(x)
0 < b < 1
(b, 1)
(1, 0)
x
VA
x = 0
Logarithmic
Decay
0< b < 1

*Sketch. State the domain and range in interval notation.*

$(0, \infty)$ $(-\infty, \infty)$

*a)* $y = \log_2 x$

$2 > 1$ **Growth**

$x > 0$ domain

$x = 0$ VA

Change equation to exponential form:

$y = \log_2 x$ (plug in for y)

$2^y = x$ (Set the exponent equal to zero)

| x | y |
|---|---|
| 1/2 | −1 |
| 1 | 0 * |
| 2 | 1 |

$y = 0$

$2^0 = x$

$1 = x$

$y = 1$

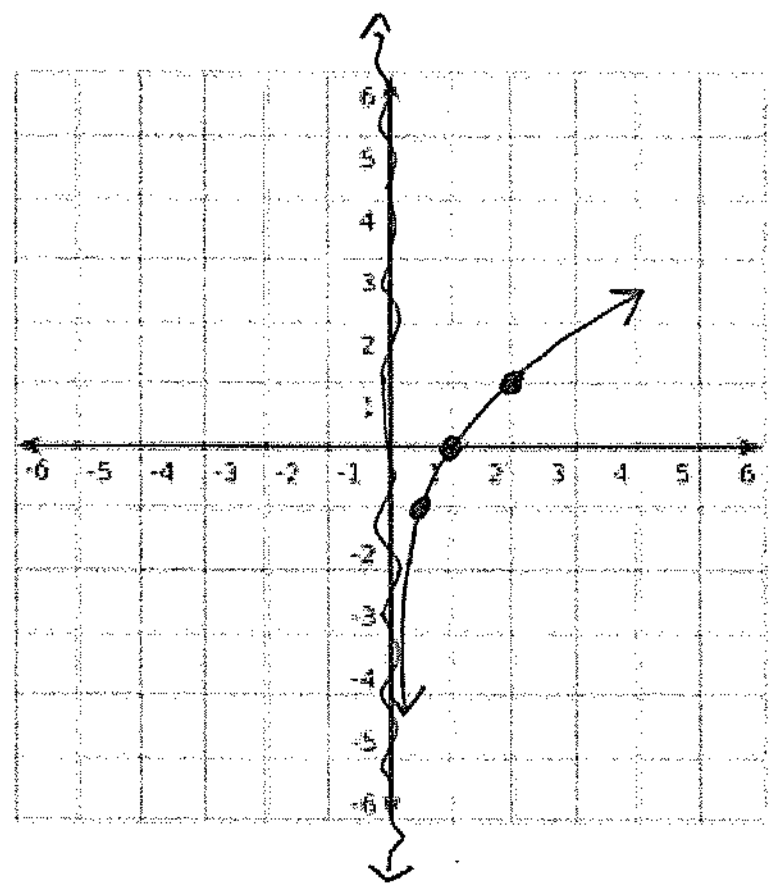
$2^1 = x$

$2 = x$

$y = -1$

$2^{-1} = x$

$\frac{1}{2} = x$




$x = 0$ VA

***Sketch. State the domain and range in interval notation.***

$(2,\infty)$ $(-\infty,\infty)$

*b)* $y = \log_3(x-2)$

$3>1$ ↗

Growth

$x-2>0$

$x>2$ domain

↓

$x=2$ VA

change to exponential form ...

$y = \log_3(x-2)$

$3^y = x-2$

$3^y + 2 = x$ ← plug in

so... $x = 3^y + 2$

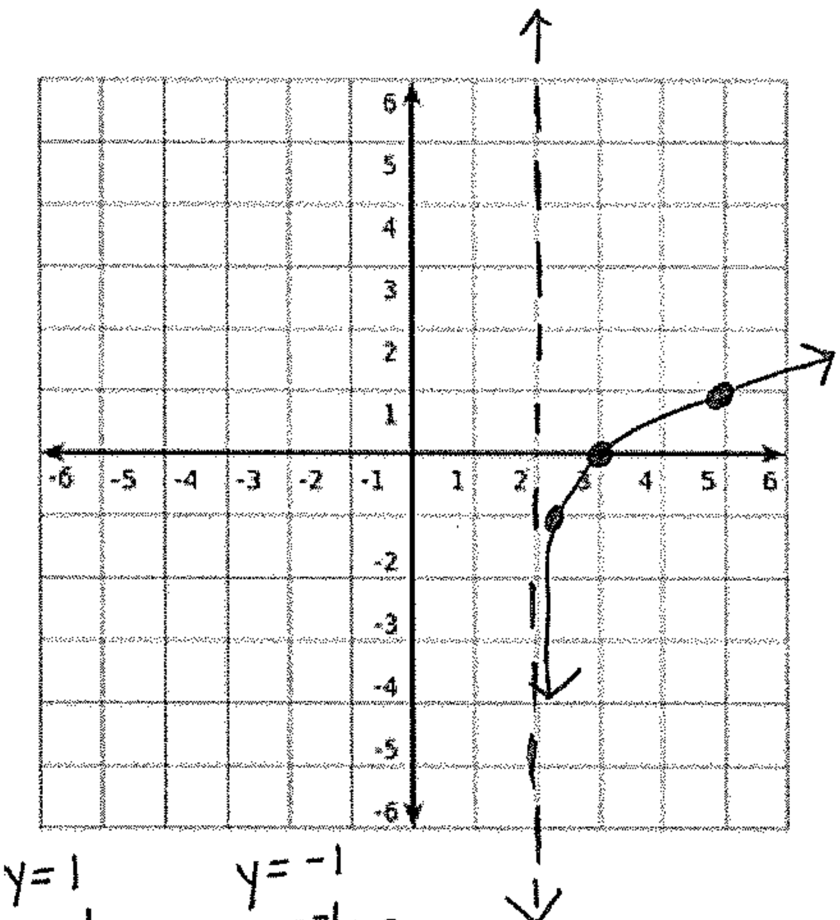
plug in

| $x$ | $y$ |
|---|---|
| $2\frac{1}{3}$ | $-1$ |
| $3$ | $0$ * |
| $5$ | $1$ |

$y=0$
$x=3^0+2$
$x=1+2$
$x=3$

$y=1$
$x=3^1+2$
$x=3+2$
$x=5$

$y=-1$
$x=3^{-1}+2$
$x=\frac{1}{3}+2$
$x=2\frac{1}{3}$




$x=2$
VA

***Sketch. State the domain and range in interval notation.***

$(0, \infty)$ $(-\infty, \infty)$

c) $y = 1 + \log_4 x$ or $y = \log_4 x + 1$ ← no ( ) here.

$4 > 1$ Growth

$x > 0$ domain

$x = 0$ VA

change to exponential form:

Isolate the "log" and make sure it's positive

$y = 1 + \log_4 x$

$-1 \quad -1$

add ( ) → $(y-1) = \log_4 x$

$4^{(y-1)} = x$

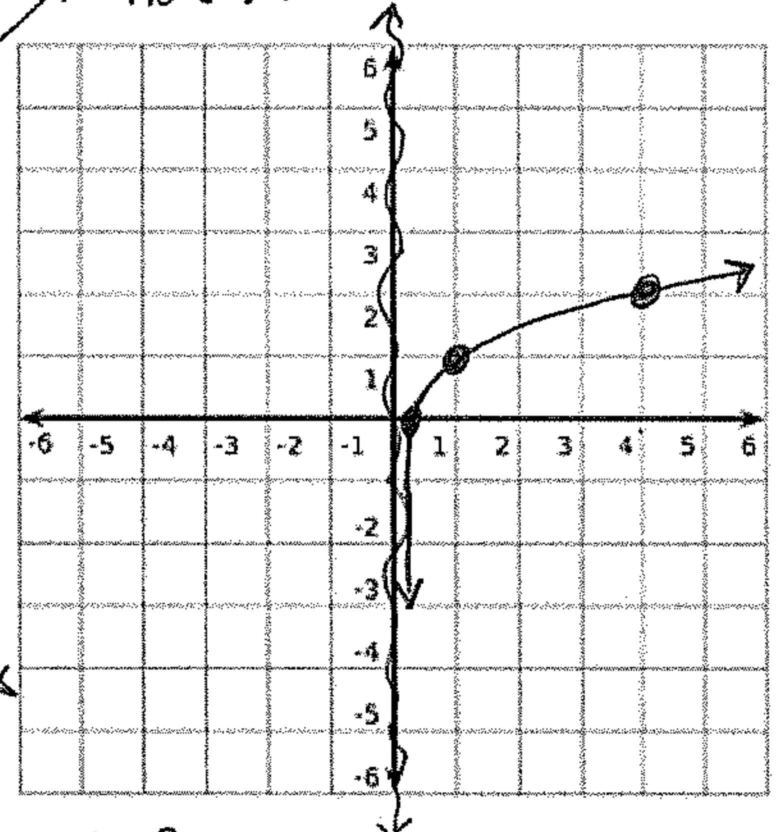
reorder: $x = 4^{(y-1)}$ ← plug in for y, set = 0

| x | y |
|---|---|
| 1/4 | 0 |
| 1 | 1 ★ |
| 4 | 2 |

$y - 1 = 0$
$y = 1$ ★
$x = 4^{(1-1)}$
$x = 4^0$
$x = 1$

$y = 2$
$x = 4^{(2-1)}$
$x = 4^1$
$x = 4$

$y = 0$
$x = 4^{(0-1)}$
$x = 4^{-1}$
$x = 1/4$



*Sketch. State the domain and range in interval notation.*

$(-3, \infty)$ $(-\infty, \infty)$

*d)* $y = \log_2(x+3) - 1$

$2 > 1$

**Growth**

$x + 3 > 0$

$x > -3$ domain

$x = -3$ VA

change to exponential form:

$y = \log_2(x+3) - 1$

$+1 \qquad +1$

Isolate the "log"

add ( )

$(y+1) = \log_2(x+3)$

$2^{(y+1)} = x + 3$

$-3 \qquad -3$

$2^{(y+1)} - 3 = x$

Reorder:

$x = 2^{(y+1)} - 3$

| x | y |
|---|---|
| $-2\frac{1}{2}$ | $-2$ |
| $-2$ | $-1$ |
| $-1$ | $0$ |

$y + 1 = 0$

$y = -1$

$\star\ x = 2^{(-1+1)} - 3 = 2^0 - 3 = 1 - 3 = -2$

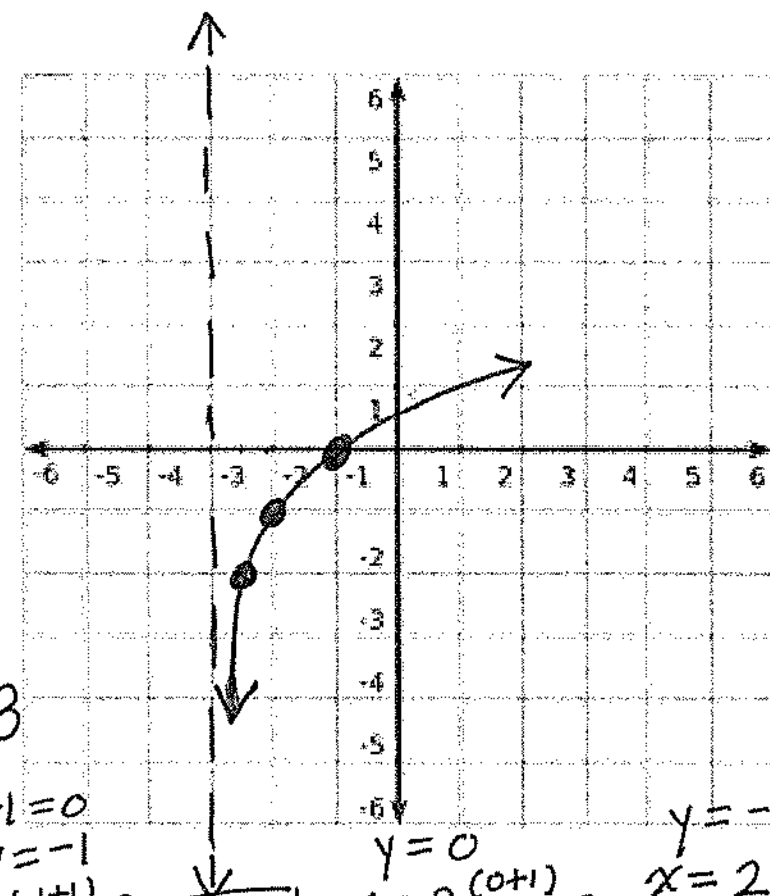
$x = -3$ VA

$y = 0$

$x = 2^{(0+1)} - 3 = 2^1 - 3 = -1$

$y = -2$

$x = 2^{(-2+1)} - 3 = 2^{-1} - 3 = \frac{1}{2} - 3 = -2\frac{1}{2}$



*Sketch. State the domain and range in interval notation.* $(0, \infty)$ $(-\infty, \infty)$

e) $y = -\log_2 x$ ← $x > 0$ domain

$2 > 1$ (base) — Growth

$x = 0$ VA

change to exponential form:

$y = -\log_2 x$

$-y = \log_2 x$ ← must be a positive isolated log

$2^{(-y)} = x$

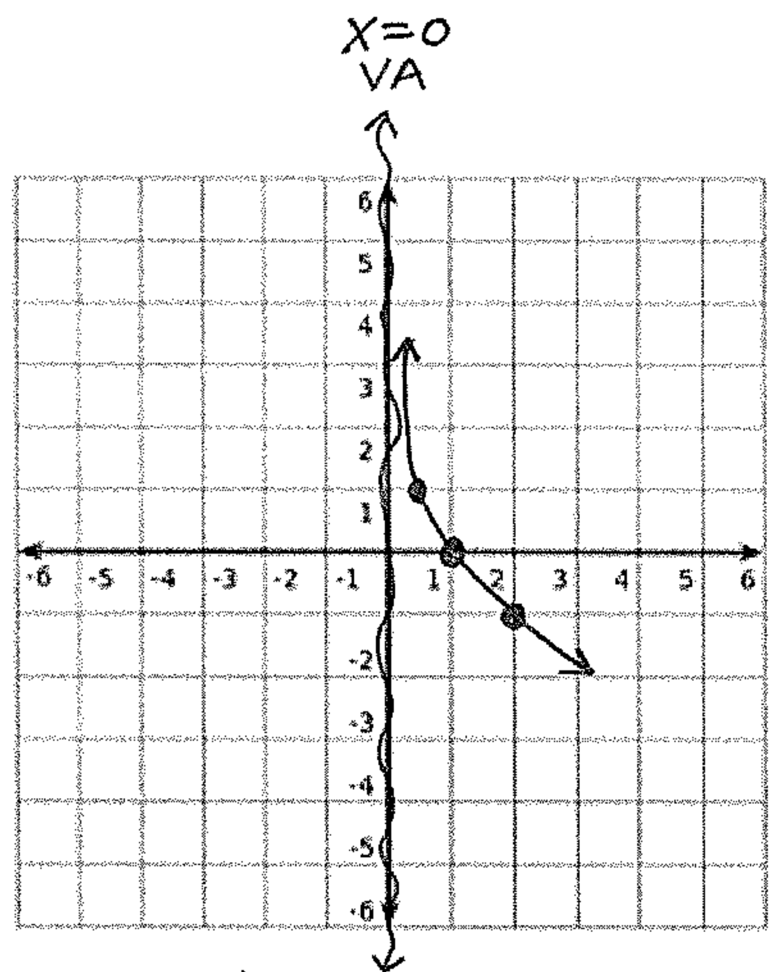
reorder: $x = 2^{(-y)}$

| x | y |
|---|---|
| 2 | -1 |
| 1 | 0 ★ |
| ½ | 1 |

$y = 0$
$x = 2^0$
$x = 1$

$y = 1$
$x = 2^{(-1)}$
$x = \frac{1}{2}$

$y = -1$
$x = 2^{(-(-1))}$
$x = 2^1$
$x = 2$



*Sketch. State the domain and range in interval notation.*

$(-3, \infty)$ $(-\infty, \infty)$

*f)* $y = \log_{1/2}(x+3)$

$0 < \frac{1}{2} < 1$

Decay

$x + 3 > 0$

$x > -3$ domain

$x = -3$ VA

change to exponential form:

$y = \log_{1/2}(x+3)$

$\left(\frac{1}{2}\right)^y = x + 3$

$-3 \qquad -3$

add ( ) on fractions

$\left(\frac{1}{2}\right)^y - 3 = x$

reorder: $x = \left(\frac{1}{2}\right)^y - 3$ plug in

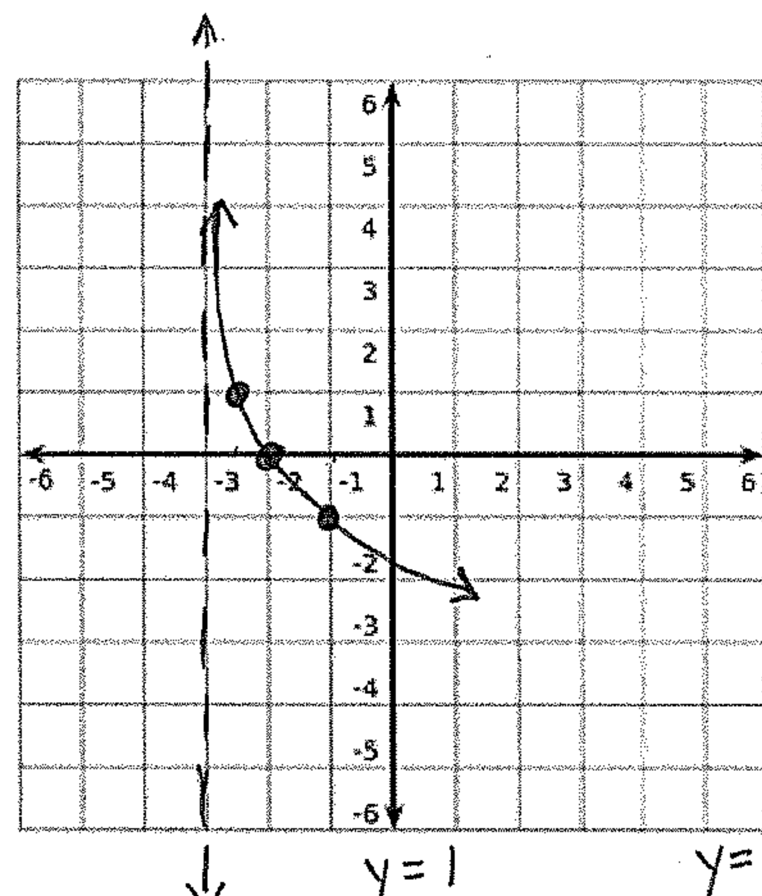
| x | y |
|---|---|
| -1 | -1 |
| -2 | 0 * |
| $-2\frac{1}{2}$ | 1 |

$y = 0$

$x = (1/2)^0 - 3$

$x = 1 - 3$

$x = -2$



$x = -3$ VA

$y = 1$

$x = (1/2)^1 - 3$

$x = 1/2 - 3$

$x = -2\frac{1}{2}$

$y = -1$

$x = \left(\frac{1}{2}\right)^{-1} - 3$

$x = \left(\frac{2}{1}\right)^1 - 3$

$x = 2 - 3 = -1$

# Solving Exponential Equations

Types of Exponential Equations:

1. $a^x = b$, where a and b are integral powers of the same number

   ex: $27^x = 9$

2. $a^x = b$, where a and b are NOT integral powers of the same number

   ex: $3^x = 5$

Type 1

**Property of Equality for Exponential Equations**

If $a^x = a^y$, then $x = y$.

To solve these equations, use the property of equality to make the bases equal.

ex: Solve.

a) $3^x = 9^{x+2}$

- Get the same base.

$3^x = (3^2)^{(x+2)}$ ← add ( )

- same base on each side, set exponents equal

solve for $x$.

$x = 2(x+2)$

$x = 2x + 4$

$-2x \quad -2x$

$-x = 4$

$\boxed{x = -4}$

ex: Solve.

b) $27^x = 81^{x-2}$

$$(3^3)^x = (3^4)^{(x-2)}$$

add

$$3x = 4(x-2)$$

$$3x = 4x - 8$$

$$-4x \quad -4x$$

$$-x = -8$$

$$\boxed{x = 8}$$

ex: Solve.

c) $7^{5-x} = \left(\frac{1}{7}\right)^{5x+3}$

add ↓ ↙ add

$7^{(5-x)} = (7^{-1})^{(5x+3)}$

$5 - x = -1(5x+3)$

$5 - x = -5x - 3$

$-5 + 5x \qquad +5x - 5$

$\frac{4x}{4} = \frac{-8}{4}$

$x = -2$

ex: Solve.

d) $125^x = \left(\frac{1}{25}\right)^{x-1}$

$\frac{1}{25} \Rightarrow \frac{1}{5^2} \Rightarrow 5^{-2}$

$(5^3)^x = (5^{-2})^{(x-1)}$ add

$3x = -2(x-1)$

$3x = -2x + 2$

$+2x \quad +2x$

$\frac{5x}{5} = \frac{2}{5}$

$x = \frac{2}{5}$

ex: Solve.

e) $4^{3x-1} = 2^{3x-7}$

$$(2^2)^{(3x-1)} = 2^{(3x-7)} \quad \leftarrow \text{add}$$

$$2(3x-1) = 3x-7$$

$$6x-2 = 3x-7$$
$$-3x+2 \quad -3x+2$$

$$\frac{3x}{3} = \frac{-5}{3}$$

$$\boxed{x = \frac{-5}{3}}$$

ex: Solve.

f) $32^{-x-4} = 16^{x-2}$

$$(2^5)^{(-x-4)} = (2^4)^{(x-2)}$$ ← add

$$5(-x-4) = 4(x-2)$$

$$-5x-20 = 4x-8$$

$$-4x+20 \quad -4x+20$$

$$\frac{-9x}{-9} = \frac{12}{-9}$$

$$x = -\frac{12}{9}$$

$$\boxed{x = -\frac{4}{3}}$$

## ex: Solve.

★ g) $3^x \cdot 9^{x+1} = 3^{x-1}$

$$3^x \cdot (3^2)^{(x+1)} = 3^{(x-1)}$$

same base

so... add the exponents.

★ can not cancel bases until there is only one base on each side.

$$3^{x+2(x+1)} = 3^{(x-1)}$$

• now cancel bases

$$x + 2(x+1) = x - 1$$

$$x + 2x + 2 = x - 1$$

$$3x + 2 = x - 1$$
$$-x \quad -2 \qquad -x \quad -2$$

$$\frac{2x}{2} = \frac{-3}{2}$$

$$\boxed{x = -\frac{3}{2}}$$

Review Concept:

$$a^2 \cdot a^3 = a^5$$

same base — add exponents

ex: Solve.

* h) $2^{x^2-x} = 4^{3-x}$

$$2^{(x^2-x)} = (2^2)^{(3-x)}$$

$$x^2 - x = 2(3-x)$$

$$x^2 - x = 6 - 2x$$

$$+2x \quad\quad +2x$$

$$x^2 + x = 6$$

$$-6 \quad\quad -6$$

Quadratic (not linear)

set equation = 0

factor?

SR? (no middle term)

QF?

no gcf → $x^2 + x - 6 = 0$ Factor?

$-6$: $1 \cdot 6$; $x \cdot x$; $-2$, $+3$ ✓

$$(x-2)(x+3) = 0$$

$x - 2 = 0$ | $x + 3 = 0$

$\boxed{x = 2}$ | $\boxed{x = -3}$