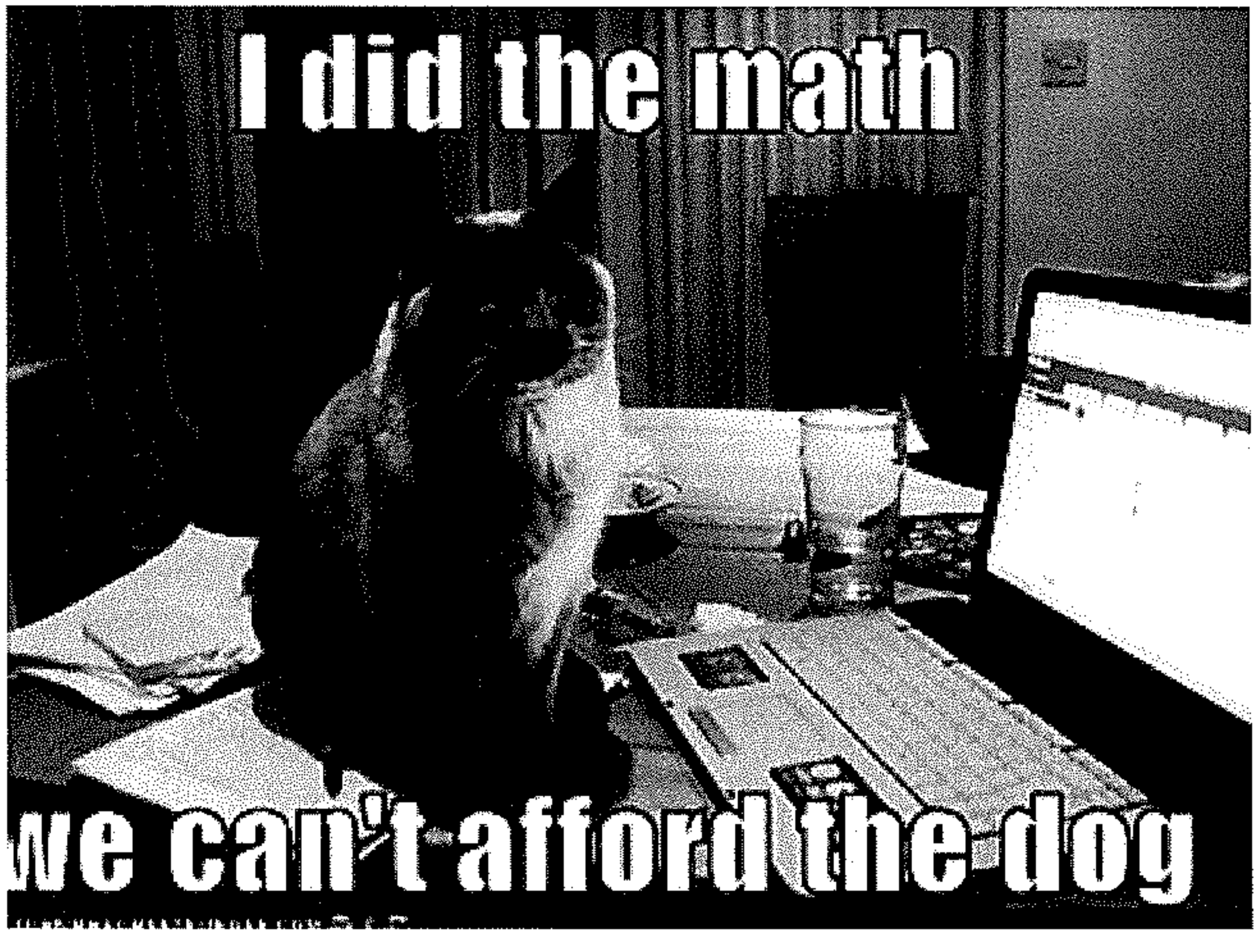
A2: Evaluating Logarithms

Notes



ex: Evaluate.

a) $2^5 = \boxed{32}$

b) $81^{3/4} = \left(\sqrt[4]{81}\right)^3$ $\qquad 3^4 = 81$

$= (3)^3 = \boxed{27}$

c) $9^{-5/2} = \dfrac{1}{9^{5/2}} = \dfrac{1}{(\sqrt{9})^5} = \dfrac{1}{(3)^5} = \boxed{\dfrac{1}{243}}$

d) $-16^{5/4} = -1 \cdot 16^{5/4}$

no ( )

$= -1 \cdot \left(\sqrt[4]{16}\right)^5$

$= -1 \cdot (2)^5$

$= -1 \cdot 32$

$= \boxed{-32}$

ex: Solve by guess and check.

a) $2^x = 16$

$\boxed{x=4}$

b) $3^x = \frac{1}{3}$

$\boxed{x=-1}$

c) $71^x = 1$

$\boxed{x=0}$

d) $25^x = 5$

$x = \frac{1}{2}$ (for square root)

e) $27^x = 9$

$x = \frac{2}{3}$ (for cube root of 27, squared)

$27^{2/3} \Rightarrow \left(\sqrt[3]{27}\right)^2 \Rightarrow (3)^2$

$\Rightarrow 9$

Definition of a Logarithm

Let b and y be positive numbers with $b \neq 1$. The **logarithm of y with base b** is denoted by $\log_b y$ and is defined as follows:

Logarithmic form　　　　　　　　　　Exponential form

$$\underbrace{\log_b}_{\text{base}} y = x \quad \text{if and only if} \quad \underbrace{b}_{\text{base}}^{x} = y$$

The expression $\log_b y$ is read as "log base b of y."

Changing btwn forms

equals

$\log_b y = x$

base

raised to

$b^x = y$

ex: Rewrite in exponential form.

a) $\log_{3} 9 = 2$

$\boxed{3^2 = 9}$

b) $\log_{22} 1 = 0$

$\boxed{22^0 = 1}$

ex: Rewrite in logarithmic form.

a) $\underset{\text{base}}{3}^5 = 243$

Check:

$\log_3 243 = 5$

$$\boxed{\log_3 243 = 5}$$

b) $\underset{\text{base}}{27}^{-2/3} = \frac{1}{9}$

$$\boxed{\log_{27}\left(\frac{1}{9}\right) = -\frac{2}{3}}$$

ex: Evaluate.

a) $\log_4 64 = \boxed{3}$

$\log_{4^{1}}(4^{3}) = \frac{3}{1}$

b) $\log_3 81 = \boxed{4}$

$\log_{3^{1}}(3^{4}) = \frac{4}{1}$

ex: Evaluate.

c) $\log_5 25 = \boxed{2}$

$\log_{5^1}(5^2) = \frac{2}{1}$

d) $\log_7\left(\frac{1}{7}\right) = \boxed{-1}$

$\log_{7^1}(7^{-1}) = \frac{-1}{1}$

ex: Evaluate.

e) $\log_{13} 1 = \boxed{0}$

$13^0 = 1$

If the argument of the log is 1, the log must equal 0.

f) $\log_{25} 5 = \boxed{\frac{1}{2}}$

$\log_{5^2}(5^1) = \frac{1}{2}$

ex: Evaluate.

g) $\log_5\left(\frac{1}{125}\right) = \boxed{-3}$

$\frac{1}{125} \Rightarrow \frac{1}{5^3} = 5^{-3}$

$\log_{5^1}(5^{-3}) = \frac{-3}{1}$

h) $\log_{81} 27 = \boxed{\frac{3}{4}}$

$3^3 = 27$

$3^4 = 81$

$\log_{3^4}(3^3) = \frac{3}{4}$

ex: Evaluate.

*i) $\log_2(-4) = \boxed{\text{not possible}}$

The argument of a log can not be negative or zero.

j) $\log_{25}\left(\frac{1}{5}\right) = \boxed{\frac{-1}{2}}$

$\log_{5^{\boxed{2}}}(5^{-1}) = \frac{-1}{2}$

Special Logarithms

SPECIAL LOGARITHMS A **common logarithm** is a logarithm with base 10. It is denoted by $\log_{10}$ or simply by log. A **natural logarithm** is a logarithm with base e. It can be denoted by $\log_e$, but is more often denoted by ln.

Common Logarithm

$$\log_{10} x = \log x$$

The 10 is not usually written for the base.

Natural Logarithm

$$\log_e x = \underline{\ln} x$$

loge is usually condensed to ln

Most calculators have keys for evaluating common and natural logarithms.

ex: Evaluate.

a) $\log 100 = \boxed{2}$

$\log_{10^{1}}(10^{2}) = \frac{2}{1}$

b) $\log\left(\frac{1}{10}\right) = \boxed{-1}$

$\log_{10^{1}}(10^{-1}) = \frac{-1}{1}$

ex: Evaluate.

c) $\log .001 = \boxed{-3}$

tenths hundredths thousandths

$.001 = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$

$\log_{10}\left(\frac{1}{1000}\right)$

$\log_{10}(10^{-3}) = \frac{-3}{1}$

d) $\ln 1 = \boxed{0}$

$\log_e 1 = 0$

$e^0 = 1$

ex: Evaluate.

e) $\ln\left(\frac{1}{e}\right) = \boxed{-1}$

$\log_e\left(\frac{1}{e}\right)$

$\log_{e^1}\left(e^{-1}\right) = \frac{-1}{1}$

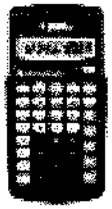
f) $\ln e^2 = \boxed{2}$

$\log_{e^1}\left(e^{2}\right) = \frac{2}{1}$

ex: Evaluate.

g) $\ln e = \boxed{1}$

$\log_{e^1} e^1 = \frac{1}{1}$



ex: Evaluate on your calculator. Round to 3 decimal places.

a) $\log 16 \approx \boxed{1.204}$

b) $\ln 7 \approx \boxed{1.946}$