Analyzing Polynomial Functions
Factoring Review

Notes

REVIEW

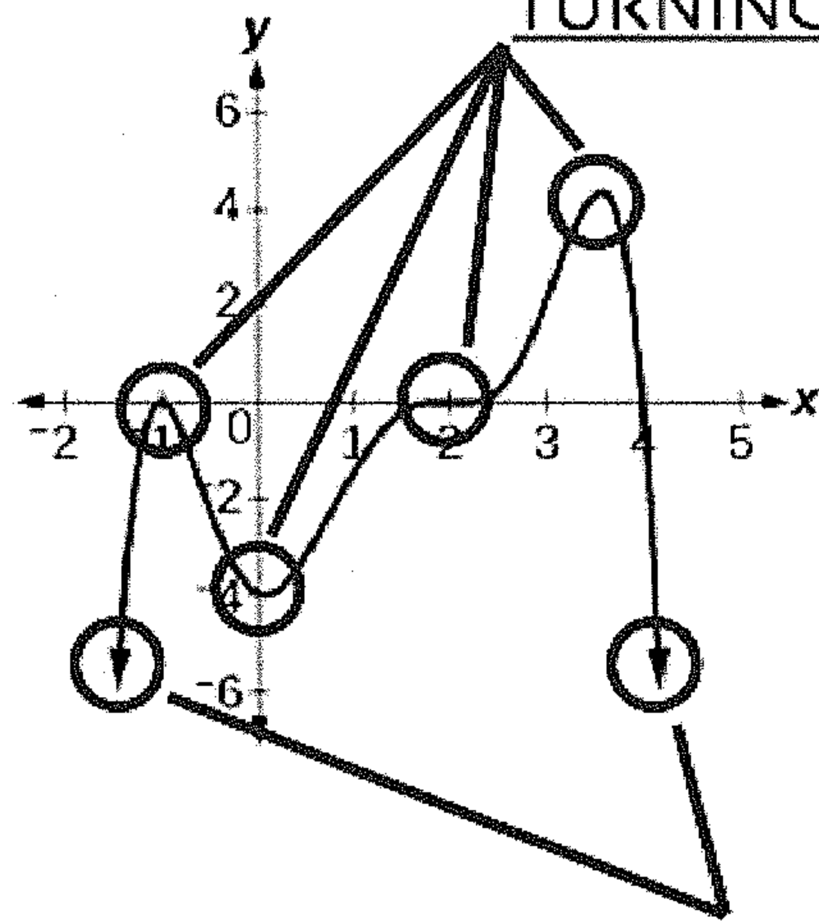
What is a polynomial function?

A polynomial function is an expression involving one or more monomials. Polynomial functions have variables with whole exponents, real coefficients and contain no division by variables. The degree of a polynomial is the largest exponent (attached to a variable). The leading coefficient is the coefficient of the term that defines the degree.

$$f(x) = 2x^2 - 4x + \frac{1}{7} \qquad f(x) = 0$$

$$f(x) = \frac{x^3}{5} \qquad f(x) = 3x^5 - x^4 + 5x - 1$$

Graphs of Polynomial Functions



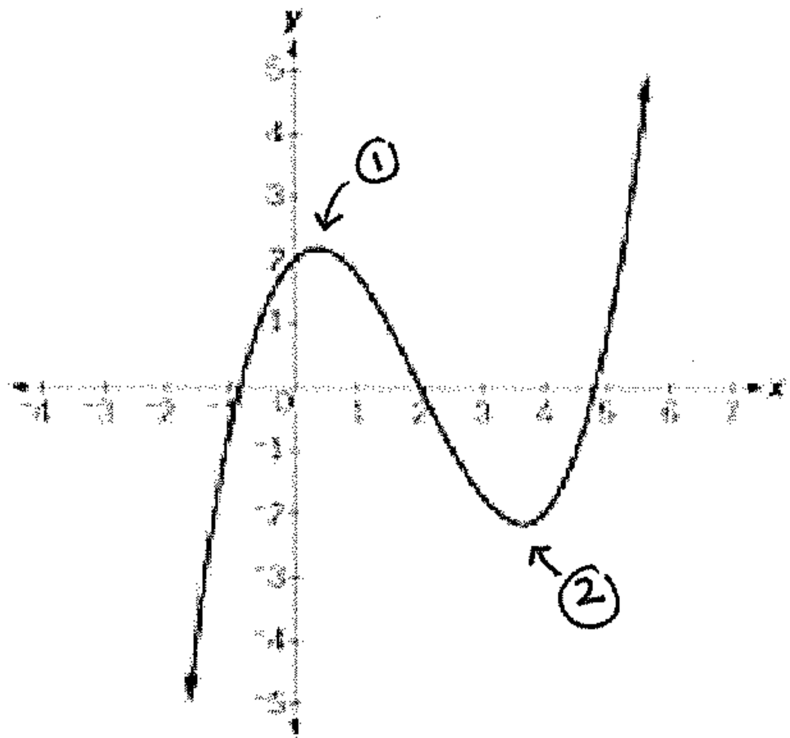
<u>TURNING POINTS</u>

Shows <u>END BEHAVIOR</u>

*A turning point can occur at a maximum, minimum or at a "flat point."

ex: Using the graph determine the number of turning points.

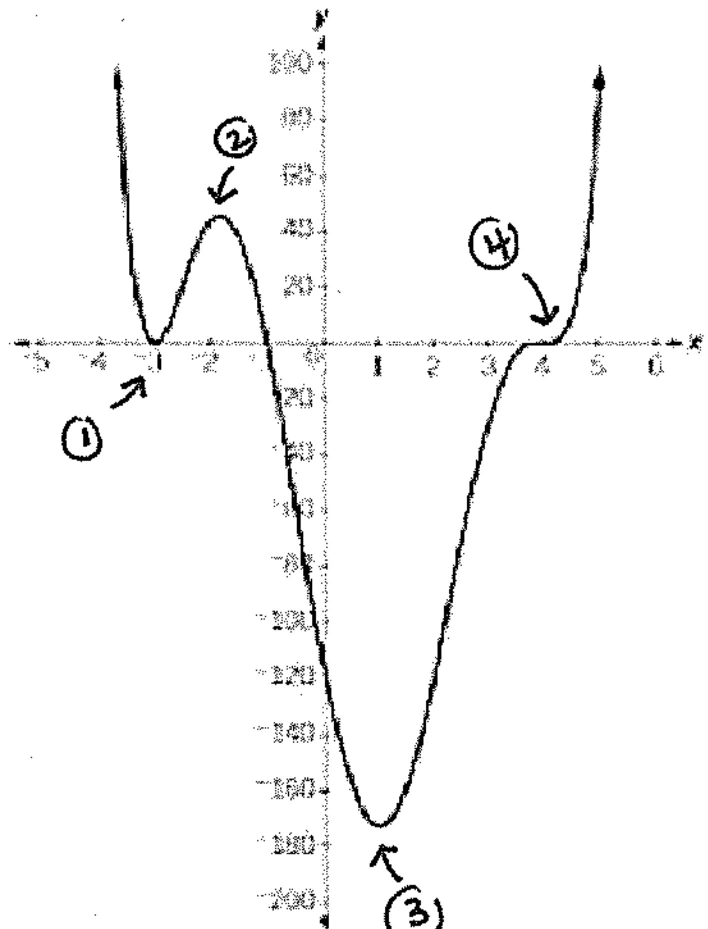
a)



$\boxed{\text{2 turns}}$

ex: Using the graph determine the number of turning points.

b)



$\boxed{\text{4 turns}}$

Polynomial Degrees and Number of Turning Points

Polynomial Type	Degree	Maximum Number of Turning Points
Constant	0	0
Linear	1	0
Quadratic	2	1
Cubic	3	2
n^{th} Degree Polynomial	n (≥ 4)	$n-1$

ex: Determine the degree and state the maximum number of turning points.

a) $f(x) = 2x^3 + 5x^2 - 9$

$\boxed{\text{Degree 3}}$

$\boxed{\text{Max turns : 2}}$

b) $f(x) = 9x^4 - 6x^5$

$\boxed{\text{Degree 5}}$

$\boxed{\text{Max turns : 4}}$

ex: Determine the degree and state the maximum number of turning points.

c) $f(x) = (3\underline{\underline{x}} - 1)^2 \longrightarrow x^2$

$\boxed{\text{Degree } 2}$

$\boxed{\text{max turns : } 1}$

d) $f(x) = (x^2 - 5)(2x + 7)^3$

$x^2 \cdot (x)^3$

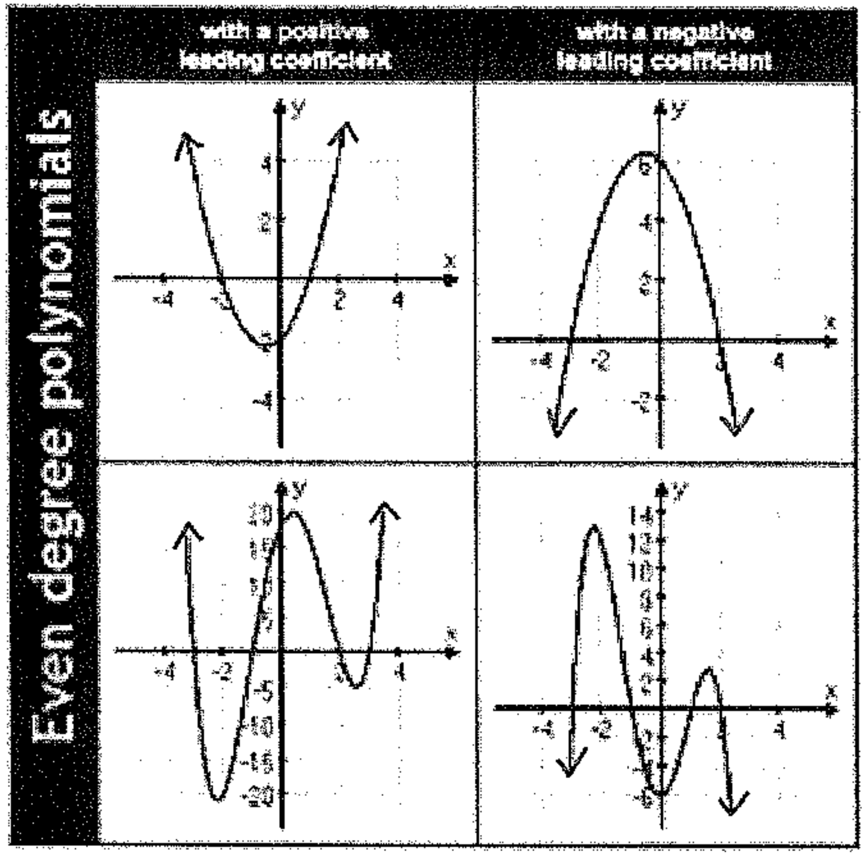
$x^2 \cdot x^3$

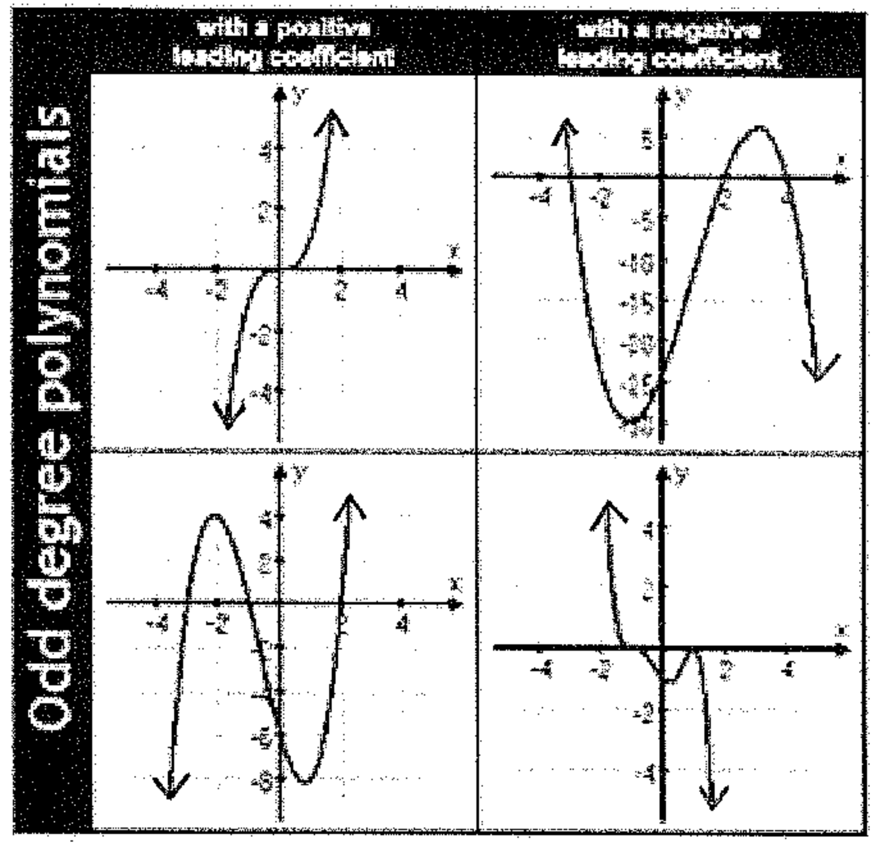
x^5

$\boxed{\text{Degree } 5}$

$\boxed{\text{max turns : } 4}$

End Behavior, Degrees & Leading Coefficients
"EB"





Even: ↑ ↑ (LC ⊕) or ↓ ↓ (LC ⊖)

Odd: ↓ ↑ (LC ⊕) or ↑ ↓ (LC ⊖)

Stating End Behavior

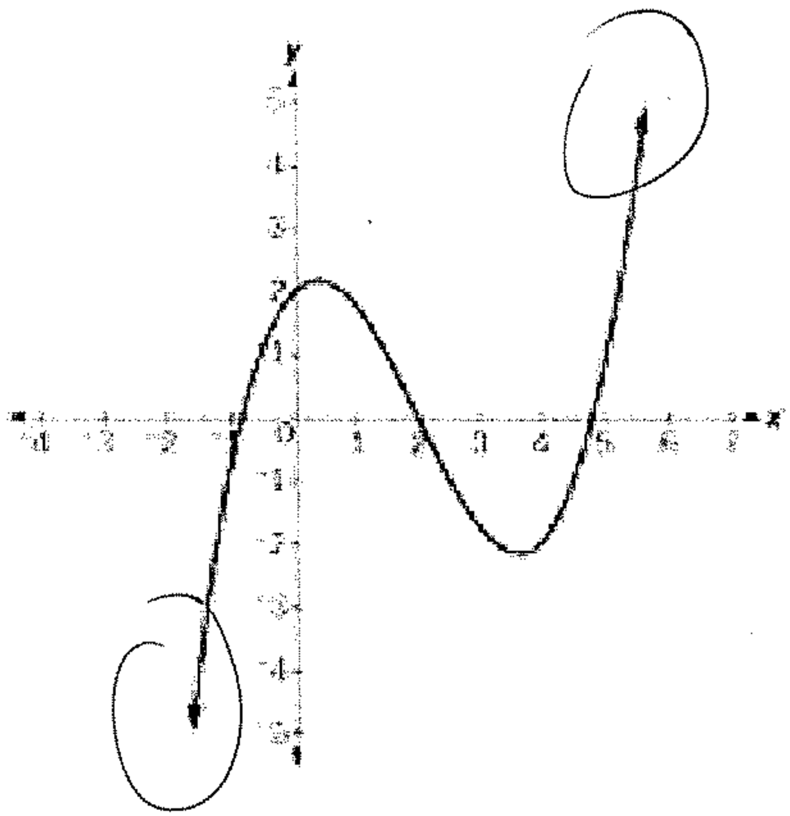
$x \to -\infty, \quad y \to$ ______

$x \to \infty, \quad y \to$ ______

answer for a polynomial will always be $-\infty$ or ∞.

ex: Determine the end behavior of each polynomial.

a)

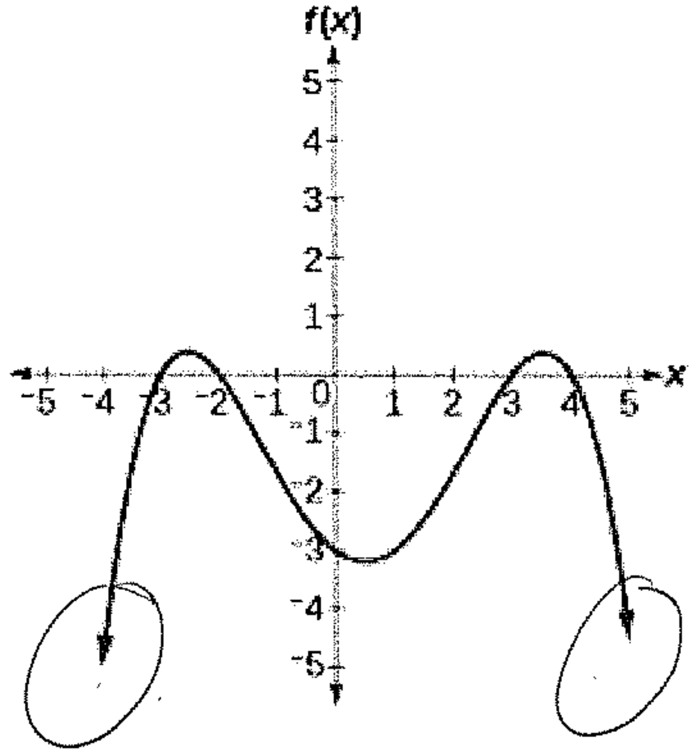


$x \to -\infty, \ y \to -\infty$

$x \to \infty, \ y \to \infty$

ex: Determine the end behavior of each polynomial.

b)



$$x \to -\infty,\ y \to \underline{\underline{-\infty}}$$

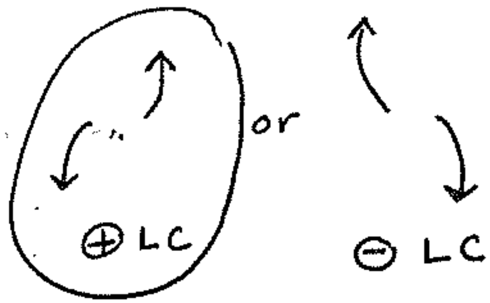
$$x \to \infty,\ y \to \underline{\underline{-\infty}}$$

EB

ex: Determine the end behavior of each polynomial.

c) $f(x) = 2x^{3} + 5x^2 - 9$

Degree 3
(odd)



LC: 2
positive

EB

EB: $\boxed{\begin{array}{l} x \to -\infty, \; y \to -\infty \\ x \to \infty, \; y \to \infty \end{array}}$

EB

ex: Determine the end behavior of each polynomial.

d) $f(x) = 9x^4 \underline{-6}x^{5}$

Degree 5
(odd)

$\oplus$ LC or $\ominus$ LC

EB

LC: -6
(negative)

EB: $\boxed{\begin{array}{l} x \to -\infty,\ y \to \infty \\ x \to \infty,\ y \to -\infty \end{array}}$

EB

ex: Determine the end behavior of each polynomial.

e) $f(x) = (3x-1)^2 \rightarrow (3x)^2 \rightarrow \underline{\underline{9}}x^2$

$(3x-1)(3x-1)$

Degree: 2
(even)

$\uparrow \; \uparrow$ or $\swarrow \; \downarrow$

$\oplus$ LC $\quad \ominus$ LC

LC: 9
(positive) $\rightarrow$ EB

EB: $\boxed{\begin{array}{l} x \to -\infty,\ y \to \infty \\ x \to \infty,\ y \to \infty \end{array}}$

EB

ex: Determine the end behavior of each polynomial.

f) $f(x) = (x^2 - 5)^{1}(2x + 7)^{3}$

$(x^2)^1 (2x)^3$

$x^2 \cdot 2^3 \cdot x^3$

$8x^2 \cdot x^3$

$8x^5$

Degree: 5 (odd)

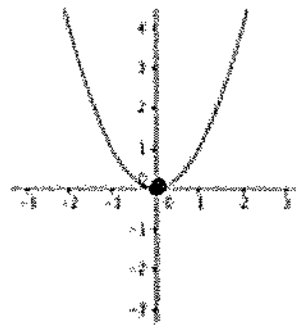
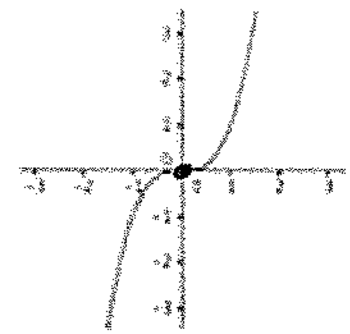
LC: 8 (positive)

$\oplus$ LC or $\ominus$ LC

EB

EB: $\boxed{\begin{array}{l} x \to -\infty,\ y \to -\infty \\ x \to \infty,\ y \to \infty \end{array}}$

Bouncing and Crossing Zeros

In the graph below the graph "bounces" off the x-axis at x=0. B	In the graph below the graph "crosses" the x-axis at x=0. C goes thru the x-axis
	

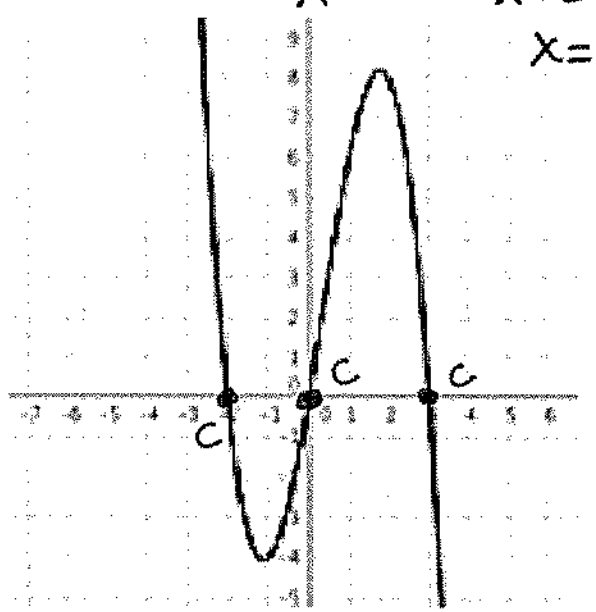
ex: Using the graph or the equation of the polynomial function,

1. Find the zeros. State the multiplicity if greater than 1.
2. Determine whether the graph "crosses" the x-axis or "bounces" off the x-axis at each zero.

a) $f(x) = -x^1(x+2)^1(x-3)^1$

$x=0$ $\quad x+2=0$ $\quad x-3=0$

$x=-2$ $\quad x=3$



x-int(s) zeros	Multiplicity	Cross or Bounce
-2	1	Cross
0	1	Cross
3	1	Cross

Odd multiplicity = cross

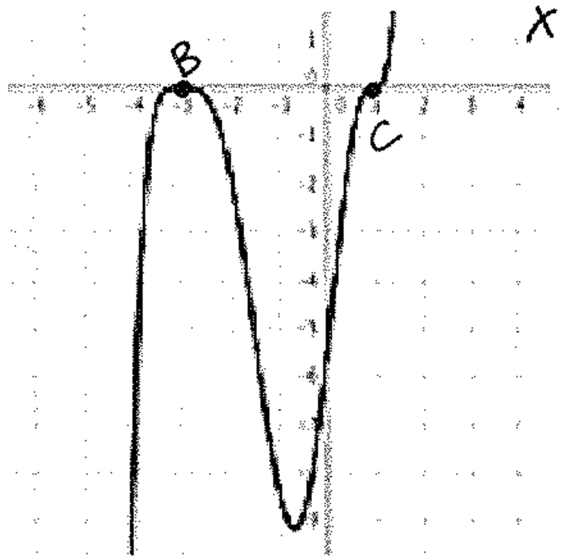
ex: Using the graph or the equation of the polynomial function,

1. Find the zeros. State the multiplicity if greater than 1.
2. Determine whether the graph "crosses" the x-axis or "bounces" off the x-axis at each zero.

b) $f(x)=\frac{1}{15}(x+3)^{4}(x-1)^{3}$

$x+3=0$
$x=-3$

$x-1=0$
$x=1$



zeros X-int	mult.	Cross/Bounce?
-3	4	Bounce
1	3	Cross

Even multiplicity = Bounce

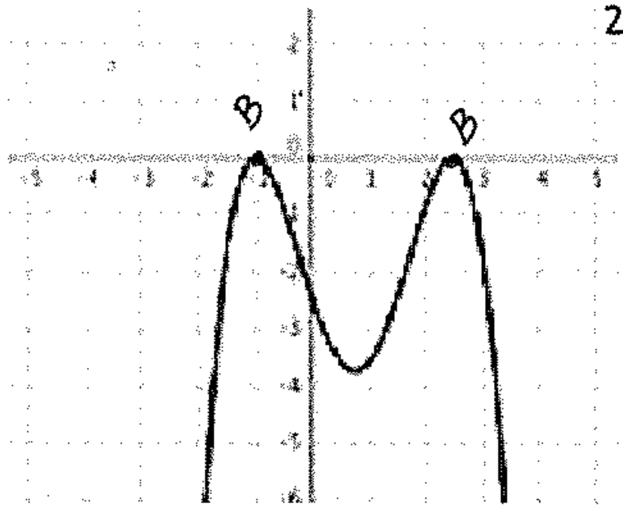
ex: Using the graph or the equation of the polynomial function,

1. Find the zeros. State the multiplicity if greater than 1.
2. Determine whether the graph "crosses" the x-axis or "bounces" off the x-axis at each zero.

c) $f(x) = -\frac{1}{10}(2x-5)^2(x+1)^2$

$2x-5=0$
$2x=5$
$x=\frac{5}{2}$

$x+1=0$
$x=-1$



zeros

X-int	mult.	Cross / Bounce?
$\frac{5}{2}$ or 2.5	2	Bounce
-1	2	Bounce

- A graph "crosses" the x-axis at a zero if the multiplicity of that zero is ___odd___.
- A graph "bounces" off the x-axis at a zero if the multiplicity of that zero is ___even___.

REVIEW

ex: Factor completely, if possible.

a) $4x^3 - 25x^2 + 25x$

gcf: $x(4x^2 - 25x + 25)$

$$\boxed{x(4x-5)(x-5)}$$

$+100$

$\cancel{2}$ $\cancel{50}$

$\cancel{4}$ $\cancel{25}$

$\frac{4x}{-5}$ $\frac{4x}{-20}$

$\div 4$

$\frac{1x}{-5}$

REVIEW

ex: Factor completely, if possible.

b) $x^2 + 9$

no gcf

sum of squares

$\boxed{\text{Prime}}$

REVIEW

ex: Factor completely, if possible.

no gcf for all 4 terms

c) $2x^3 - 3x^2 + 10x - 15$

$x^2 \quad x^2 \quad 5 \quad 5$

$x^2(2x-3)+5(2x-3)$

$$\boxed{(2x-3)(x^2+5)}$$

REVIEW

ex: Factor completely, if possible.

d) $8x^3 - 1$ cubes (use the rule) $a = 2x$ $a^2 = 2x \cdot 2x = 4x^2$

no gcf $b = 1$ $ab = 2x \cdot 1 = 2x$

S O AP $b^2 = 1$

$= \boxed{(2x - 1)(4x^2 + 2x + 1)}$

REVIEW

ex: Factor completely, if possible.

e) $\frac{2x^2}{2} - \frac{32}{2}$

gcf: $2(x^2 - 16)$

DOS

$\boxed{2(x+4)(x-4)}$

REVIEW

ex: Factor completely, if possible.

f) $\frac{-5x^2}{-1} + \frac{18x}{-1} - \frac{9}{-1}$

factor out the negative!!!

$-(5x^2 - 18x + 9)$

$+45$

$\frac{5x}{-3}$ $\frac{5x}{-15}$

$\div 5$

$\frac{1x}{-3}$

$\boxed{-(5x-3)(x-3)}$

must have.

REVIEW

ex: Factor completely, if possible.

double

g) $2x^{4} + 7x^{2} + 6$

no gcf

$$\boxed{(2x^2+3)(x^2+2)}$$

+12

$\dfrac{2x^{2}}{+3}$ $\dfrac{2x^{2}}{+4}$

÷ 2

$\dfrac{1x^{2}}{+2}$

REVIEW

ex: Factor completely, if possible.

no gcf for all 4 terms

h) $x^5 - x^3 + 64x^2 - 64$

$x^3(x^2-1) + 64(x^2-1)$

$(x^2-1)(x^3+64)$

DOS cubes

$a = x$

$b = 4$

$a^2 = x^2$

$ab = 4x$

$b^2 = 16$

S O AP

$\boxed{(x+1)(x-1)(x+4)(x^2-4x+16)}$

REVIEW

ex: Factor completely, if possible.

double

i) $x^4 + 4x^2 + 5$

no gcf

$+5$

$-1 \quad +5$

no!

Does not factor.

$\boxed{\text{Prime}}$