

A2: Adding and Subtracting Rational Expressions

Adding or Subtracting Rational Expressions

Step 1 Identify a common denominator.

Step 2 Multiply each expression by an appropriate form of $\frac{1}{1}$ so that each term has the common denominator as its denominator.

Step 3 Write each expression using the common denominator.

Step 4 Add or subtract the numerators, combining like terms as needed.

Step 5 Factor as needed.

Step 6 Simplify as needed.

Perform the indicated operation. Simplify if possible.

$$a) \frac{2}{5} + \frac{1}{3}$$

Least common multiple
(LCM)

Factors? 5 3

$$\frac{2(3)}{5(3)} + \frac{1(5)}{3(5)}$$

(15)

$$\frac{6}{15} + \frac{5}{15}$$

$$\frac{6+5}{15} = \boxed{\frac{11}{15}}$$

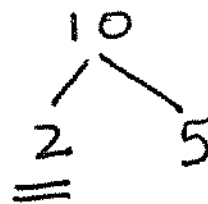
Perform the indicated operation. Simplify if possible.

$$b) \frac{1}{2} - \frac{3}{10}$$

LCM \rightarrow LCD ^{"denominator"}

factors

2



$$\frac{\underline{1}(5)}{\underline{2}(5)} - \frac{3}{10}$$

$$\frac{5}{10} - \frac{3}{10}$$

LCM: 2 \cdot 5

(10)

$$\frac{5-3}{10} = \frac{2}{10} = \boxed{\frac{1}{5}}$$

Perform the indicated operation. Simplify if possible.

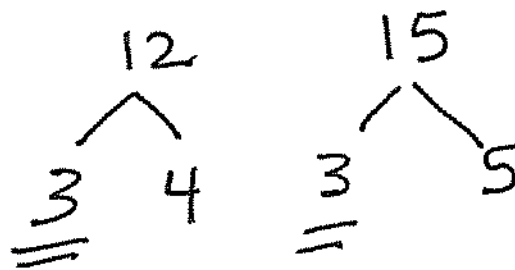
$$c) \quad \frac{5}{12} + \frac{4}{15}$$

$$\frac{5(5)}{12(5)} + \frac{4(4)}{15(4)}$$

$$\frac{25}{60} + \frac{16}{60}$$

$$\frac{25+16}{60} = \boxed{\frac{41}{60}}$$

LCM?



$$\text{LCM: } \underline{3} \cdot 4 \cdot 5$$

(60)

Perform the indicated operation. Simplify if possible.

$$d) \frac{2}{x} + \frac{x}{3}$$

LCM?

$$x \quad 3$$

$x \neq 0$

$$\frac{2(3)}{x(3)} + \frac{x(x)}{3(x)}$$

$$(3x)$$

$$\frac{6}{3x} + \frac{x^2}{3x}$$

reorder:

$$\frac{6+x^2}{3x} = \boxed{\frac{x^2+6}{3x}}$$

Perform the indicated operation. Simplify if possible.

e)

$$\frac{1}{2x} - \frac{3}{5x^2} \leftarrow \boxed{x \neq 0}$$

$$\frac{1(5x)}{2x(5x)} - \frac{3(2)}{5x^2(2)}$$

$$\frac{5x}{10x^2} - \frac{6}{10x^2}$$

$$= \boxed{\frac{5x-6}{10x^2}}$$

LCM?

$$\underline{\underline{2x}} \quad \underline{\underline{5x^2}}$$
$$5x \cdot \underline{\underline{x}}$$

$$\underline{\underline{x}} \cdot 2 \cdot 5x$$

$$\boxed{10x^2}$$

Perform the indicated operation. Simplify if possible.

f) $\frac{5}{2x} + \frac{1}{3x}$ ← $\boxed{x \neq 0}$ LCM? $\underline{\underline{2x}}$ $\underline{\underline{3x}}$

$\frac{5(3)}{2x(3)} + \frac{1(2)}{3x(2)}$ $\underline{\underline{x}} \cdot 2 \cdot 3$

$\frac{15}{6x} + \frac{2}{6x}$ $\textcircled{6x}$

$= \frac{15+2}{6x} = \boxed{\frac{17}{6x}}$

Perform the indicated operation. Simplify if possible.

$$a) \frac{4x-5}{3x+1} + \frac{x+4}{3x+1}$$

$$\frac{\text{LCM:}}{3x+1}$$

$$\frac{4x-5 + x+4}{3x+1}$$

now
add like
terms in
numerator

$$\boxed{\frac{5x-1}{3x+1}}$$

$$3x+1=0$$

$$3x=-1$$

$$\boxed{x \neq -\frac{1}{3}}$$

Perform the indicated operation. Simplify if possible.

$$b) \frac{2x+7}{x^2+2x-15} - \frac{x+10}{x^2+2x-15}$$

LCM:

$$x^2+2x-15$$

Careful!

$$\frac{2x+7 - (x+10)}{x^2+2x-15}$$

• now distribute the negative

$$\frac{2x+7 - x - 10}{x^2+2x-15}$$

• add like terms in the numerator

$$\frac{x-3}{x^2+2x-15}$$

$$x^2+2x-15$$

← Factor to see if it can be simplified.

$$\frac{x-3}{(x+5)(x-3)}$$

$$\begin{array}{cc} -15 & \leftarrow \\ 1 & 15 \\ -3 & +5 \end{array}$$

$$x \neq -5, 3$$

$$\frac{1}{x+5}$$

Perform the indicated operation. Simplify if possible.

$$c) \frac{7}{x+2} + \frac{3}{2x+4}$$

LCM:

$$\underline{\underline{(x+2)}} \quad 2x+4 \\ \underline{\underline{2(x+2)}}$$

$$\frac{(2)}{(2)} \cdot \frac{7}{(x+2)} + \frac{3}{2(x+2)}$$

LCM: $\underline{\underline{(x+2)}} \cdot 2$

$$\frac{14}{2(x+2)} + \frac{3}{2(x+2)}$$

$$\boxed{2(x+2)}$$

$$\frac{14+3}{2(x+2)}$$

$$\boxed{\frac{17}{2(x+2)}}$$

$$\boxed{x \neq -2}$$

Perform the indicated operation. Simplify if possible.

d) $\frac{7}{2x+3} - \frac{3}{x+5}$

LCM:

$(2x+3)(x+5)$

Be careful

$\frac{7(x+5)}{(2x+3)(x+5)} - \frac{3(2x+3)}{(x+5)(2x+3)}$

LCM: $(2x+3)(x+5)$

$\frac{7x+35}{(2x+3)(x+5)} + \frac{-6x-9}{(2x+3)(x+5)}$ ← add like terms

$\frac{x+26}{(2x+3)(x+5)}$

$x \neq -\frac{3}{2}, -5$

Perform the indicated operation. Simplify if possible.

e) $\frac{x+3}{x^2-x-2} + \frac{x}{x-2}$

LCM:

x^2-x-2 $(x-2)$

$(x-2)(x+1)$

$\frac{(x+3)}{(x-2)(x+1)} + \frac{x}{(x-2)(x+1)}$

LCM: $(x-2)(x+1)$

$\frac{1x+3}{(x-2)(x+1)} + \frac{x^2+1x}{(x-2)(x+1)}$

• add like terms

$$\frac{x^2+2x+3}{(x-2)(x+1)}$$

~~$\frac{x+3}{-1+x}$~~

$$x \neq 2, -1$$

Perform the indicated operation. Simplify if possible.

f) $\frac{x}{x^2 - 4} - \frac{10x}{x - 2}$

$\frac{x}{x^2 - 4}$
DOS

x

Beware!

$\frac{10x}{(x-2)(x+2)}$

$(x+2)(x-2)$

LCM:

$x^2 - 4$
DOS

$(x-2)$

LCM: $(x-2)(x+2)$

$\frac{1x}{(x+2)(x-2)} + \frac{-10x^2 - 20x}{(x+2)(x-2)}$

$\frac{-10x^2 - 19x}{(x+2)(x-2)}$

$\frac{-x(10x+19)}{(x+2)(x-2)}$

$x \neq -2, 2$

Review:

Simplify and state the excluded values.

$$a) \frac{\sqrt{x^3 - 2x^2 + x - 2}}{x^4 + x^2 - 2}$$

$$\frac{x^2(x-2) + 1(x-2)}{(x^2-1)(x^2+2)}$$

$$\begin{array}{cc} -2 & \\ \underline{x^2} & \underline{x^2} \\ -1 & +2 \end{array} \leftarrow$$

$$\frac{(x-2)(x^2+1)}{(x+1)(x-1)(x^2+2)}$$

$$\boxed{\frac{(x-2)(x^2+1)}{(x+1)(x-1)(x^2+2)}}$$

$$\boxed{x \neq -1, 1}$$

nothing cancels!

$$x^2 + 2 = 0$$

$$\sqrt{x^2} = \sqrt{-2}$$

$$|x| = i\sqrt{2}$$

$$x = \pm i\sqrt{2}$$

↑
not a restriction
b/c imaginary

Simplify and state the excluded values.

$$b) \frac{x^2 - 9}{x^3 + 3x} \leftarrow \text{DOS}$$
$$\leftarrow \text{GCF}$$

$$\frac{(x+3)(x-3)}{x(x^2+3)} \leftarrow \boxed{x \neq 0}$$

nothing cancels!

$$x^2 + 3 = 0$$

$$\sqrt{x^2} = \sqrt{-3}$$

$$|x| = i\sqrt{3}$$

$$x = \pm i\sqrt{3}$$

Perform the indicated operation and simplify.

$$c) \frac{\overset{\text{POS}}{4x^2-1}}{\underset{\text{DOS}}{x^2-4}} \cdot \frac{x-2}{2x-1}$$

$$\frac{(2x+1)\cancel{(2x-1)}}{(x+2)\cancel{(x-2)}} \cdot \frac{\cancel{(x-2)}}{\cancel{(2x-1)}} \left| \leftarrow x \neq -2, 2, \frac{1}{2} \right|$$

$$\boxed{\frac{2x+1}{x+2}}$$

$$\begin{aligned} 2x-1 &= 0 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

Perform the indicated operation and simplify.

$$d) \frac{x^2 - 4}{2x^2 - 5x + 2} \div \frac{2x^2 - 3x - 2}{4x^2 - 1}$$

keep change flip

$$\text{DOS } \frac{x^2 - 4}{2x^2 - 5x + 2} \cdot \frac{4x^2 - 1}{2x^2 - 3x - 2}$$

$$\frac{2x}{-1} \begin{matrix} +4 \\ \frac{2x}{-4} \end{matrix}$$

$$\frac{2x}{+1} \begin{matrix} -4 \\ \frac{2x}{-4} \end{matrix}$$

$$\frac{(x+2)(x-2)}{(2x-1)(x-2)} \cdot \frac{(2x+1)(2x-1)}{(2x+1)(x-2)}$$

$$= \boxed{\frac{x+2}{x-2}} \quad \boxed{x \neq \frac{1}{2}, 2, -\frac{1}{2}}$$